SOCY7708: Hierarchical Linear Modeling
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Class notes: HLM Model Building Strategies

The issue of centering

An important decision that we make when conducting HLM analysis is whether and how you'd like to center your predictors. Here, we will discuss the issues involved in making these decisions.

Level-1 predictors:

1. Natural metric (X):
You should only use the original metric if the value of 0 for a predictor is a meaningful value (i.e., actually exists in the data). When 0 is not meaningful, the estimate of the intercept will be arbitrary and may be estimated with poor precision. Lack of precision in HLM can be very problematic. First, because you are estimating within-group intercepts, thus with possibly small N, the estimates may be quite unstable. Second, because you may be trying to model variation in these intercepts, your model will be affected by the unreliability of the estimates.

2. Grand-mean centering (X - grand mean):
This will address the problems with estimation of intercept in original metric. Because the 0 values will fall in the middle of the distribution of the predictors, the intercepts will be estimated with much more precision. The intercept is also interpretable. Specifically, if all predictors are grand mean centered, it will represent the value for a person in an average level 2 group with a (grand) average on every predictor. The interpretation of the intercepts is now "adjusted group mean." The interpretation of slopes does not change. E.g., our measure of SES is already grand-mean centered because it is a standardized scale. So we can interpret the fixed effect for the intercept as the average math achievement adjusted for SES – i.e., the average math achievement for someone with average SES.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(FEMALE_{ij}) + \beta_{2j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + \gamma_{21}(SECTOR_j)$$

```
. sum ses

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
         ses |      7,185    .0001434    .7793552     -3.758      2.692

. gen ses_m=ses-r(mean)

. sum ses_m
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
       ses_m |      7,185   -7.90e-09    .7793552  -3.758143   2.691857

. mixed mathach c.ses_m##i.sector i.female##i.sector  || id: female, cov(unstr)

Performing EM optimization ...

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     680.89
Log likelihood = -23254.764                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+--------------------------------------------------------------
         ses_m |   2.922083   .1399094    20.89   0.000     2.647865     3.1963
      1.sector |   2.085121   .4060651     5.13   0.000     1.289248    2.880994
               |
sector#c.ses_m |
             1 |  -1.292315   .2107619    -6.13   0.000    -1.705401   -.8792293
               |
      1.female |  -1.222337   .2312292    -5.29   0.000    -1.675538    -.769136
               |
 female#sector |
          1 1  |   .0298036    .389324     0.08   0.939    -.7332574    .7928646
               |
         _cons |   12.43798   .2611924    47.62   0.000     11.92605    12.94991
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(female) |   1.050623   .5973015      .3447634    3.201644
                  var(_cons) |   4.115261   .7097072      2.934955    5.770234
           cov(female,_cons) |   -1.14854    .544171     -2.215095   -.0819842
-----------------------------+------------------------------------------------
               var(Residual) |   36.44462   .6201845      35.24913    37.68066
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 307.69                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov, corr

Random-effects correlation matrix for level id

             |    female      _cons
-------------+----------------------
      female |         1
       _cons |  -.552362          1
```

Note that while it may seem inappropriate at first to center a dummy variable, in HLM it actually can be useful. If uncentered, the intercept in a model with a dummy variable is the average value when the dummy variable is 0. If the dummy variable is centered, the intercept then becomes the mean adjusted for the proportion of cases with the dummy variable=1. For example, if the indicator for gender variable is centered around the grand mean, this centered predictor can take two values. If the subject is female, it will equal the proportion of male students in the sample. If the subject is male, it will equal to minus the proportion of female students in the sample. Zero on this variable becomes the average proportion of female students. The intercept again will be the adjusted group mean – in this case, it is adjusted for the difference among level-2 units in the percentage of female students.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(FEMALE_{ij} - \overline{FEMALE}_{..}) + \beta_{2j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}(SECTOR_j)$$

```
. sum female

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
      female |      7,185    .5281837    .4992398          0          1

. gen female_m=female-r(mean)

. sum female_m

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
    female_m |      7,185    1.68e-09    .4992398  -.5281837   .4718163

. mixed mathach c.ses_m##i.sector i.female_m##i.sector  || id: female_m, cov(unstr)
female_m:  factor variables may not contain noninteger values
r(452);

. mixed mathach c.ses_m##i.sector c.female_m##i.sector  || id: female_m, cov(unstr)

Performing EM optimization ...

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     680.89
Log likelihood = -23254.764                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
         mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+------------------------------------------------------------
           ses_m |   2.922084   .1399094    20.89   0.000     2.647867    3.196302
        1.sector |   2.100861   .3246478     6.47   0.000     1.464563    2.737159
                 |
  sector#c.ses_m |
              1  |  -1.292317   .2107619    -6.13   0.000    -1.705403   -.8792314
                 |
        female_m |  -1.222338   .2312331    -5.29   0.000    -1.675547   -.7691294
                 |
sector#c.female_m |
              1  |   .0297957   .3893304     0.08   0.939    -.7332779    .7928692
                 |
           _cons |   11.79236   .2158534    54.63   0.000      11.3693    12.21543
------------------------------------------------------------------------------
```

```
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
               var(female_m) |   1.050782   .5972948      .3448775    3.201549
                  var(_cons) |   3.195066   .4878502      2.368706    4.309716
         cov(female_m,_cons) |  -.5936354   .3779508     -1.334405    .1471347
-----------------------------+------------------------------------------------
               var(Residual) |    36.4446   .6201836      35.24911    37.68063
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 307.69                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov, corr

Random-effects correlation matrix for level id

             |  female_m      _cons
-------------+----------------------
    female_m |         1
       _cons |  -.323984          1
```

<u>3. Group-mean centering (X – group mean):</u>
Predictors can also be centered around the mean value for the group to which they belong. The intercept can then be interpreted as the average outcome for each group. This allows interpretation of parameter estimates as person-level effects within each group (i.e. if you differ from your group's average by one unit, your math achievement will increase by X units).

Again, we can group-mean center dummy variables as well. For females, we will get a value equal to the proportion of male students in school j; for males, it will take the value equal to minus the proportion of females in that school. The fact that it is a dummy variable does not change the interpretation of the intercept when group mean-centering is employed.

Use egen command to generate an aggregated variable containing group means, then subtract group means from the original variable:

```
. bysort id: egen meanses2=mean(ses)

. gen ses_gm=ses-meanses2

. bysort id: egen meanfemale=mean(female)

. gen female_gm=female-meanfemale
```

LEVEL 1 MODEL

$$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{FEMALE}_{ij} - \overline{\text{FEMALE}}_{\bullet j}) + \beta_{2j}(\text{SES}_{ij} - \overline{\text{SES}}_{\bullet j}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{SECTOR}_j) + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + \gamma_{21}(\text{SECTOR}_j)$$

```
. mixed mathach c.ses_gm##i.sector c.female_gm##i.sector  || id: female_gm, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     529.31
Log likelihood = -23299.553                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------------
           mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+----------------------------------------------------------------
            ses_gm |   2.732804   .1444167    18.92   0.000     2.449752    3.015855
          1.sector |   2.804132   .4367607     6.42   0.000     1.948097    3.660168
                   |
   sector#c.ses_gm |
                1  |  -1.310776   .2178402    -6.02   0.000    -1.737735   -.8838173
                   |
         female_gm |  -1.224759   .2253235    -5.44   0.000    -1.666385   -.7831325
                   |
sector#c.female_gm |
                1  |   .4206202   .4105511     1.02   0.306    -.3840451    1.225286
                   |
             _cons |   11.39348   .2911603    39.13   0.000     10.82282    11.96415
------------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
              var(female_gm) |   .7984596   .5522106      .2058573    3.096989
                  var(_cons) |   6.660512   .8506139       5.18562    8.554892
        cov(female_gm,_cons) |  -.6228692   .5725962     -1.745137    .4993987
-----------------------------+------------------------------------------------
               var(Residual) |   36.44168   .6202232      35.24611     37.6778
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 786.22                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov, corr

Random-effects correlation matrix for level id

             | female_gm      _cons
-------------+----------------------
   female_gm |         1
       _cons |  -.270095          1
```

<u>Important:</u>

Under grand-mean centering or no centering, the parameter estimates reflect a combination of (1) person-level effects and (2) compositional effects. But when we use a group-centered predictor, we only estimate the person-level effects.

In order not to discard the compositional effects with group-mean centering, level-2 variables should be created to represent the group mean values for each group-mean centered predictor. Because the group mean is effectively removed from the individual scores, the level-2 values will be orthogonal to the level-1 values. E.g. we can use group mean centering for SES and using mean SES as a school level variable (here, meanses is already in the dataset, but we also created meanses2):

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + \gamma_{02}(MEANSES_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + \gamma_{12}(MEANSES_j) + u_{1j}$$

```
. mixed mathach c.ses_gm##i.sector c.ses_gm##c.meanses  || id: ses_gm, cov(unstr)
note: ses_gm omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     761.63
Log likelihood = -23248.215                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------------
           mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+----------------------------------------------------------------
            ses_gm |    2.93939   .1534841    19.15   0.000     2.638567    3.240214
          1.sector |   1.226736   .3032663     4.05   0.000     .6323451    1.821127
                   |
   sector#c.ses_gm |
                1  |  -1.643914   .2373424    -6.93   0.000    -2.109097   -1.178732
                   |
            ses_gm |          0  (omitted)
           meanses |   5.331706   .3655557    14.59   0.000      4.61523    6.048182
                   |
c.ses_gm#c.meanses |   1.042444   .2960172     3.52   0.000     .4622613    1.622627
                   |
             _cons |   12.09601   .1968495    61.45   0.000     11.71019    12.48183
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
               var(ses_gm) |   .0650065    .208139      .0001223    34.54217
                var(_cons) |   2.316889   .3607765      1.707495     3.14377
          cov(ses_gm,_cons) |   .1881343   .1983402     -.2006054    .5768741
-----------------------------+------------------------------------------------
              var(Residual) |   36.72119   .6261882      35.51417    37.96924
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 216.68                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov, corr

Random-effects correlation matrix for level id

             |    ses_gm      _cons
-------------+----------------------
      ses_gm |         1
       _cons |  .4847716          1
```

Here, the effects of SES turn out to be quite complex: For those who are in a public school whose SES is at their school's average and whose school itself is average in terms of its SES, the math achievement is 12.1. If you are in a Catholic school with such properties, it's 12.1+1.2=13.3. But if your school's average SES is 1 unit higher that the average for all schools, then your math achievement increases by 5.33. Further, in addition to these school-level effects, your individual SES also plays a role – if you are in an average (in terms of SES) public school, one unit increase in your SES will raise your math score by 2.94. In a Catholic school, that effect would be 2.94-1.64=1.30. But if you are in a public school and your school is 1 unit above an average school in its SES, then your personal SES impact (per one unit) would be 2.94+1.04=3.98. For a Catholic school in that situation, that effect of SES would become 2.94-1.64+1.04=2.34. Interestingly, personal SES seems to have stronger impact on math achievement in those schools that have relatively high school-level SES.

The choice between grand-mean centering and group-mean centering depends on your theoretical thinking about processes. If you think that the absolute values of level 1 variable matter, then use grand-mean centering. If you think that it is the relative position of the person with regards to their group's mean is what matters, then use group-centering. Importantly, you do not need to use group mean centering in order to use level 2 aggregated variables, such as meanses:

```
. mixed mathach c.ses_m##i.sector c.ses_m##c.meanses  || id: ses_m, cov(unstr)
note: ses_m omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     775.99
Log likelihood = -23248.852                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------------
          mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
------------------+-----------------------------------------------------------------
            ses_m |   2.904552   .1481728    19.60   0.000     2.614139    3.194965
         1.sector |   1.194948   .3047013     3.92   0.000     .5977443    1.792151
                  |
   sector#c.ses_m |
               1  |   -1.57687   .2242443    -7.03   0.000    -2.016381   -1.137359
                  |
            ses_m |          0  (omitted)
          meanses |   3.319093   .3847275     8.63   0.000     2.565041    4.073145
                  |
c.ses_m#c.meanses |   .8421218   .2713517     3.10   0.002     .3102822    1.373961
                  |
            _cons |    12.0959   .2007449    60.26   0.000     11.70245    12.48935
------------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(ses_m) |    .014443   .0298477      .0002515    .8293538
                 var(_cons) |   2.339569    .365851      1.721983    3.178651
            cov(ses_m,_cons) |   .1838214   .1918724     -.1922416    .5598844
-----------------------------+------------------------------------------------
               var(Residual) |    36.7444   .6202666      35.54859    37.98044
```

```
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 213.73                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov, corr

Random-effects correlation matrix for level id

             |     ses_m      _cons
-------------+----------------------
       ses_m |         1
       _cons |         1          1
```

Level-2 predictors:

Centering issues for level-2 predictors are essentially the same issues faced in any regression. If the value of 0 for a predictor is not meaningful, the intercept will not have a meaningful interpretation and the estimate may lack precision. When these conditions exist, grand mean centering is advisable. Again, if you'd like, you can center dichotomous variables as well in order to interpret the intercept as a truly average case, adjusted for all predictors.

LEVEL 1 MODEL

$$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij} - \overline{\text{SES}}_{\bullet\bullet}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j - \overline{\text{SECTOR}}_{\bullet}) + \gamma_{02}(\text{MEANSES}_j - \overline{\text{MEANSES}}_{\bullet}) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{SECTOR}_j - \overline{\text{SECTOR}}_{\bullet}) + \gamma_{12}(\text{MEANSES}_j - \overline{\text{MEANSES}}_{\bullet}) + u_{1j}$$

```
. mixed mathach c.ses_m##c.sector_m c.ses_m##c.meanses_m  || id: ses_m, cov(unstr)
note: ses_m omitted because of collinearity.

Mixed-effects ML regression                         Number of obs     =      7,185
Group variable: id                                  Number of groups  =        160
                                                    Obs per group:
                                                                  min =         14
                                                                  avg =       44.9
                                                                  max =         67
                                                    Wald chi2(5)      =     775.99
Log likelihood = -23248.852                         Prob > chi2       =     0.0000
-------------------------------------------------------------------------------------
            mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+----------------------------------------------------------------
              ses_m |    2.13215   .1093559    19.50   0.000     1.917816    2.346484
           sector_m |   1.194948   .3047013     3.92   0.000     .5977443    1.792151
                    |
 c.ses_m#c.sector_m |   -1.57687   .2242443    -7.03   0.000    -2.016381   -1.137359
                    |
              ses_m |          0  (omitted)
          meanses_m |   3.319093   .3847275     8.63   0.000     2.565041    4.073145
                    |
c.ses_m#c.meanses_m |   .8421218   .2713517     3.10   0.002     .3102822    1.373961
                    |
```

```
            _cons |   12.70552   .1485969    85.50   0.000     12.41427    12.99676
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(ses_m) |    .014443   .0298477      .0002515    .8293539
                 var(_cons) |   2.339569    .365851      1.721983    3.178651
           cov(ses_m,_cons) |   .1838214   .1918724     -.1922416    .5598844
-----------------------------+------------------------------------------------
              var(Residual) |    36.7444   .6202665      35.54859    37.98044
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 213.73                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Model Selection Strategy

To summarize, we saw that multilevel models can include 3 types of predictors:

- Level-1 predictors (e.g., student SES)
- Level-2 predictors (e.g., school SECTOR)
- Level 2 predictors that are level-1 predictors aggregated to level 2 (e.g., MEANSES)

In addition, we have a number of choices:

- The intercept can be estimated as either fixed or random (typically random)
- The effects of level 1 predictors can be estimated as either fixed effects or random effects
- Level 2 predictors can be used to predict the intercept (i.e., as direct predictors of DV)
- Level 2 predictors can explain the variation in slopes of level 1 predictors (i.e., as cross-level interactions)
- Level 1 predictors could be either grand mean centered or group mean centered

Because so many components are involved, it is best to proceed incrementally and use hypothesis testing to arrive at the most parsimonious model.

Model Development Algorithms

Two main algorithms are recommended; the first one differentiates between level 1 and level 2 variables; the second one does not.

Level-specific algorithm:

1. Fit a fully unconditional model (Model 0). Evaluate level 2 variance to see if HLM is necessary.
2. Estimate a model with random intercept and slopes using only level 1 variables and any necessary interactions among them (Model 2). Make all slopes random, unless you have substantive reasons for separating random and non-random ones. Note, however, that random slopes for interaction terms can be difficult to interpret.
3. Evaluate slope variance, decide whether some slopes should be non-random, and fix those slopes. (Do a joint significance test to doublecheck that all those slopes are jointly not significant.)

4. Based on the significance of regression coefficients, exclude variables where both coefficients and corresponding random effects are not significant. Keep the variable if the coefficient is non-significant but the random effect is. Make sure to conduct hypotheses tests to make sure these variables are jointly not significant. (Note that sometimes you might have substantive reasons to keep the variable even if its coefficient is not significant.)
5. Estimate means-as-outcomes with level 1 covariates model (Model 4) to select level 2 predictors of intercept (include both original level 2 variables and aggregates of level 1). Use hypothesis testing to trim the model.
6. For slopes with significant variance, use level 2 predictors to explain that variance (i.e., estimate an intercepts-and-slopes-as-outcomes model, Model 5). If a slope does not have significant variance but your theory suggests cross-level interaction, do include it. Use hypothesis testing to trim the model.
7. If the slope variance remaining after entering level 2 predictors is not statistically significant, estimate that slope as non-randomly varying (Model 6).

Combined algorithm:

1. Fit a fully unconditional model (Model 0). Evaluate level 2 variance to see if HLM is necessary.
2. Enter all level 2 and level 1 variables in the model, and include any within-level and cross-level interactions based on theory (Model 5). (Don't forget to use aggregates of level 1 variables.) Make all slopes random, unless you have substantive reasons for separating random and non-random ones. Note, however, that random slopes for interaction terms can be difficult to interpret.
3. Evaluate slope variance, decide whether some slopes should be non-random, and fix those slopes (Model 6). (Do a joint significance test to doublecheck that all those slopes are jointly not significant.)
4. Based on the significance of regression coefficients, exclude variables where both coefficients and corresponding random effects are not significant. Keep the variable if the coefficient is non-significant but the random effect is. Make sure to conduct hypotheses tests to make sure these variables are jointly not significant. (Note that sometimes you might have substantive reasons to keep the variable even if its coefficient is not significant.)
5. If there are remaining random slopes with significant variance, consider adding other cross-level interactions to explain that variance. If that leads to the random slope becoming non-significant, estimate that slope as non-randomly varying (Model 6).

Using Hypothesis Testing to Build Models

When making decisions what variables to include and whether to estimate random or fixed effects, we need to use hypothesis testing tools. We already saw how to do that for variance components but what about coefficients? We will do some recodes to HSB data for this example

```
. recode size (0/499=1) (500/1199=2) (1200/3000=3), gen(sized)
(7185 differences between size and sized)

. mixed mathach c.ses_m##c.sector c.ses_m##i.sized  i.female##i.sector
i.female##i.sized  || id: ses_m female, cov(unstr)
note: ses_m omitted because of collinearity.
note: 1.sector omitted because of collinearity.
```

```
Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     677.31
Log likelihood = -23246.527                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
         mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+------------------------------------------------------------
           ses_m |   3.071058   .2823452    10.88   0.000     2.517671    3.624444
          sector |    2.36977   .4358575     5.44   0.000     1.515505    3.224035
                 |
c.ses_m#c.sector |  -1.283094   .2364085    -5.43   0.000    -1.746446   -.8197415
                 |
           ses_m |          0  (omitted)
                 |
           sized |
              2  |    1.23097   .5676327     2.17   0.030     .1184298    2.343509
              3  |   1.752608    .585621     2.99   0.003     .6048122    2.900404
                 |
   sized#c.ses_m |
              2  |  -.2519453   .2923376    -0.86   0.389    -.8249165    .3210258
              3  |  -.1277427   .3097203    -0.41   0.680    -.7347833    .4792979
                 |
        1.female |  -.2816415   .4703985    -0.60   0.549    -1.203606    .6403227
        1.sector |          0  (omitted)
                 |
   female#sector |
            1 1  |  -.1739571   .4058262    -0.43   0.668    -.9693619    .6214477
                 |
    female#sized |
            1 2  |   -.712411   .5186187    -1.37   0.170    -1.728885    .3040631
            1 3  |  -1.308136   .5277861    -2.48   0.013    -2.342578   -.2736945
                 |
           _cons |   11.03738   .5302974    20.81   0.000     9.998014    12.07674
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(ses_m) |   .0864968          .            .           .
                var(female) |   .7588742          .            .           .
                 var(_cons) |   4.090261          .            .           .
          cov(ses_m,female) |  -.1480118          .            .           .
           cov(ses_m,_cons) |   .5930792          .            .           .
          cov(female,_cons) |  -.9053627          .            .           .
-----------------------------+------------------------------------------------
               var(Residual) |   36.36132          .            .           .
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 311.08                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
Warning: Standard-error calculation failed.
```

This model has problems with variance components – it is likely because SES slope variance is small and non-significant, as we discovered earlier; we will use non-randomly varying slope for SES.

```
. mixed mathach c.ses_m##c.sector c.ses_m##i.sized  i.female##i.sector
i.female##i.sized  || id: female, cov(unstr)
```

```
note: ses_m omitted because of collinearity.
note: 1.sector omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     696.49
Log likelihood = -23249.266                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
         mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+------------------------------------------------------------
           ses_m |   3.072735   .2756691    11.15   0.000     2.532434    3.613037
          sector |   2.367085   .4322866     5.48   0.000     1.519818    3.214351
                 |
c.ses_m#c.sector |  -1.276275   .2314061    -5.52   0.000    -1.729823   -.8227277
                 |
           ses_m |          0  (omitted)
                 |
           sized |
              2  |   1.276543   .5631693     2.27   0.023     .1727518    2.380335
              3  |   1.766455   .5807222     3.04   0.002     .6282603     2.90465
                 |
   sized#c.ses_m |
              2  |  -.2675328     .28581    -0.94   0.349    -.8277101    .2926445
              3  |  -.1308607   .3026749    -0.43   0.665    -.7240926    .4623711
                 |
        1.female |  -.2465819   .4707665    -0.52   0.600    -1.169267    .6761034
        1.sector |          0  (omitted)
                 |
   female#sector |
            1 1  |  -.1725857   .4063304    -0.42   0.671    -.9689787    .6238073
                 |
    female#sized |
            1 2  |  -.7773901   .5187856    -1.50   0.134    -1.794191    .2394109
            1 3  |  -1.331266   .5283803    -2.52   0.012    -2.366872   -.2956592
                 |
           _cons |   11.06016   .5258152    21.03   0.000     10.02958    12.09073
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(female) |    .768301   .5565778      .1857369    3.178079
                  var(_cons) |   3.992242   .6855878      2.851281    5.589767
          cov(female,_cons) |  -.9597241   .5169883     -1.973003    .0535544
-----------------------------+------------------------------------------------
               var(Residual) |   36.43078   .6198063      35.23601    37.66606
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 305.60                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

<u>1. Single parameter tests of significance.</u>

Single parameter tests are presented in your regular HLM output; in practice, there is no need to run such tests in addition to the regular output, but for learning purposes, we will start with these. Suppose we want to test whether a specific coefficient is zero:

```
. test 1.female=0
```

```
 ( 1)  [mathach]1.female = 0

           chi2(  1) =    0.27
         Prob > chi2 =    0.6004
```

To see how to refer to each coefficient, we look at the vector of coefficients stored in e(b):

```
. mat list e(b)

e(b)[1,29]
        mathach:     mathach:     mathach:     mathach:     mathach:     mathach:     mathach:
                                  c.ses_m#           o.          1b.           2.           3.
          ses_m       sector     c.sector        ses_m        sized        sized        sized
y1    3.0727353    2.3670847   -1.2762753            0            0    1.2765433     1.766455

        mathach:     mathach:     mathach:     mathach:     mathach:     mathach:     mathach:
       1b.sized#     2.sized#     3.sized#          0b.           1.          0b.          1o.
        co.ses_m      c.ses_m      c.ses_m       female       female       sector       sector
y1            0   -.26753279   -.13086071            0   -.24658195            0            0

        mathach:     mathach:     mathach:     mathach:     mathach:     mathach:     mathach:
      0b.female#   0b.female#   1o.female#    1.female#   0b.female#   0b.female#   0b.female#
       0b.sector    1o.sector    0b.sector     1.sector     1b.sized     2o.sized     3o.sized
y1            0            0            0   -.17258568            0            0            0

        mathach:     mathach:     mathach:     mathach:    lns1_1_1:    lns1_1_2:  atr1_1_1_2:
      1o.female#    1.female#    1.female#
        1b.sized      2.sized      3.sized        _cons        _cons        _cons        _cons
y1            0   -.77739013   -1.3312656    11.060155   -.13178684     .6921765   -.61550344

        lnsig_e:

          _cons
y1     1.797707

. test 1.female#1.sector=0

 ( 1)  [mathach]1.female#1.sector = 0

           chi2(  1) =    0.18
         Prob > chi2 =    0.6710
```

For both of these tests, we fail to reject H0 and could remove these coefficients from the model. But we often want to evaluate whether coefficients are jointly significant.

<u>2. Multi-parameter tests of significance.</u>

Here, we test the hypothesis that multiple coefficients are all equal to 0. Typically, we do that in order to decide whether they can be omitted from the model. This can either be coefficients for different variables (possibly related, e.g. sets of dummies), or coefficients for the same variable in different parts of the model. For example, for could test that all coefficients for SES slope are zero. That would mean testing a combined hypothesis:

G20=0

G21=0

G22=0

G23=0

```
. test ses_m=0

 ( 1)  [mathach]ses_m = 0
```

```
          chi2(  1) =  124.24
        Prob > chi2 =    0.0000

. test c.ses_m#c.sector, acc

 ( 1)  [mathach]ses_m = 0
 ( 2)  [mathach]c.ses_m#c.sector = 0

          chi2(  2) =  124.50
        Prob > chi2 =    0.0000

. test 2.sized#c.ses_m=0, acc

 ( 1)  [mathach]ses_m = 0
 ( 2)  [mathach]c.ses_m#c.sector = 0
 ( 3)  [mathach]2.sized#c.ses_m = 0

          chi2(  3) =  274.60
        Prob > chi2 =    0.0000

. test 3.sized#c.ses_m=0, acc

 ( 1)  [mathach]ses_m = 0
 ( 2)  [mathach]c.ses_m#c.sector = 0
 ( 3)  [mathach]2.sized#c.ses_m = 0
 ( 4)  [mathach]3.sized#c.ses_m = 0

          chi2(  4) =  543.26
        Prob > chi2 =    0.0000
```

We reject Ho; the coefficients associated with SES slope are jointly significant. But that is not surprising as some of these were individually significant. So this test is more frequently used to jointly test whether multiple variables that have non-significant coefficients can be omitted.

3. Tests for equality of coefficients.

We can also test whether two or more coefficients are equal. This is typically used when we have a series of related dummy variables, and we want to combine some dummies. We have two sized dummies here, so let's test whether they can be combined. We test:

```
. test 2.sized=3.sized

 ( 1)  [mathach]2.sized - [mathach]3.sized = 0

          chi2(  1) =    1.12
        Prob > chi2 =    0.2909

. test 2.sized#c.ses_m=3.sized#c.ses_m, acc

 ( 1)  [mathach]2.sized - [mathach]3.sized = 0
 ( 2)  [mathach]2.sized#c.ses_m - [mathach]3.sized#c.ses_m = 0

          chi2(  2) =    1.41
        Prob > chi2 =    0.4953

. test 1.female#2.sized=1.female#3.sized, acc

 ( 1)  [mathach]2.sized - [mathach]3.sized = 0
 ( 2)  [mathach]2.sized#c.ses_m - [mathach]3.sized#c.ses_m = 0
```

```
 ( 3)  [mathach]1.female#2.sized - [mathach]1.female#3.sized = 0

           chi2(  3) =    2.21
         Prob > chi2 =    0.5293
```

Here, we fail to reject Ho, so we would be able to combine those variables.

Note: If we, for example, had a set of four dummies (one omitted) and wanted to combine all of them, we would do pairwise tests for each pair, dummy 2=dummy 3, dummy 2=dummy 4, dummy 3=dummy 4.

4. Tests for variance components

If we are interested in testing hypotheses about variance components or their combinations (e.g., see step 3 in both model-building algorithms), we should utilize likelihood ratio tests and BIC values, as we learned earlier (we did that for one variance component, but that can be done for multiple ones at a time by comparing a model with them to a model without). BIC values can also be used in addition to test command to evaluate whether fixed effects parameters should be omitted.