SOCY7708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Cross-Nested Models

In this unit, we will look at cross-nested (or cross-classified or cross-effects) models. These are models where lower-level units simultaneously belong to two higher-level units—these could be two units of the same type (for examine, two neighborhoods where a person lived at different periods of time or two schools that they attended) or two units of different type (e.g., neighborhoods and schools).

For our first example, we will use data from a study of neighborhood and school contribution to educational attainment in Scotland (as used in the *Hierarchical Linear Models* textbook by Raudenbush and Bryk, 2002). We will use attain.dta file from the website; it contains data on individuals nested in neighborhoods (N=542) and schools (N=17) in such a way that people from the same neighborhood can be attending different schools, so three-level model is not possible. Here, we have to have two IDs, one for each higher-level unit – SCHID and NEIGHID.

The variables available on level 1 are:

- ATTAIN, a measure of educational attainment
- P7VRQ, denoting primary 7 verbal reasoning quotient
- P7READ, denoting primary 7 reading test scores
- DADOCC, indicating the father's occupation scaled on the Hope-Goldthorpe scale in conjunction with the Registrar General's social-class index
- DADUNEMP, an indicator for father's unemployment status (1 if unemployed, 0 otherwise)
- DADED, an indicator for father's educational level (1 if schooling past the age of 15, 0 otherwise)
- MOMED, an indicator for mother's educational level (1 if schooling past the age of 15, 0 otherwise)
- MALE, an indicator for student gender (1 if male, 0 if female)

A variable at neighborhood level is:

- DEPRIVE, an index of social deprivation for the local community where the respondent lived

We do not have any variables on school level but we could aggregate level 1 variables to create such aggregated measures.

First, how many different neighborhoods do people in specific schools come from, and how many different schools do people in a given neighborhood attend? Let's count:

```
. use attain.dta, clear

. sort neighid schid

. bysort neighid schid: gen num=_n
```

```
. gen count=(num==1)

. bysort neighid: egen schoolsinneigh=total(count)

. egen neightag=tag(neighid)

. tab schoolsinneigh if neightag==1

schoolsinne |
        igh |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |        309       58.97       58.97
          2 |        176       33.59       92.56
          3 |         33        6.30       98.85
          4 |          6        1.15      100.00
------------+-----------------------------------
      Total |        524      100.00

. bysort schid neighid: gen n=_n

. gen countn=(n==1)

. bysort schid: egen neighperschool=total(countn)

. egen schtag=tag(schid)

. tab neighperschool if schtag==1

neighpersch |
        ool |      Freq.     Percent        Cum.
------------+-----------------------------------
         11 |          1        5.88        5.88
         29 |          1        5.88       11.76
         31 |          1        5.88       17.65
         37 |          1        5.88       23.53
         40 |          1        5.88       29.41
         41 |          1        5.88       35.29
         42 |          2       11.76       47.06
         43 |          1        5.88       52.94
         46 |          1        5.88       58.82
         47 |          1        5.88       64.71
         52 |          2       11.76       76.47
         53 |          1        5.88       82.35
         61 |          1        5.88       88.24
         65 |          1        5.88       94.12
         92 |          1        5.88      100.00
------------+-----------------------------------
      Total |         17      100.00
```

To estimate a cross-nested model in Stata, we use a work-around solution creating a "fake" three level model, wherein individuals are nested in neighborhoods which are then nested in an entire set but with random effects for dummy variables for schools which have their variances constrained to the same number and covariances constrained to zero. That effectively gives us two variances, one for each level. Therefore, for schools, we use specification _all: R.schid.

```
. mixed attain  || _all: R.schid || neighid:

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
```

```
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(0)      =          .
Log likelihood = -3178.3557                     Prob > chi2       =          .
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .0753532   .0722216     1.04   0.297    -.0661987     .216905
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.schid) |    .075445   .0316491      .0331553    .1716755
-----------------------------+------------------------------------------------
neighid: Identity            |
                  var(_cons) |   .1412201   .0218651       .104257     .191288
-----------------------------+------------------------------------------------
               var(Residual) |   .7990182   .0263652      .7489788    .8524007
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 207.44                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

We could use these variance components to calculate the percentage of variance due to each level of nesting; alternatively, to get a rough sense, I will estimate two two-level models and get their ICC:

```
. mixed attain  || neighid:

Mixed-effects ML regression                     Number of obs     =      2,310
Group variable: neighid                         Number of groups  =        524

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =         16

                                                Wald chi2(0)      =          .
Log likelihood = -3207.9847                     Prob > chi2       =          .
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .0820248   .0284356     2.88   0.004     .0262921    .1377575
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
neighid: Identity            |
                  var(_cons) |   .2015382   .0257242      .1569317    .2588237
-----------------------------+------------------------------------------------
               var(Residual) |   .8043706   .0265743      .7539364    .8581785
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 148.18        Prob >= chibar2 = 0.0000

. estat icc

Intraclass correlation
```

```
------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                     neighid |   .2003543   .0223053      .1601713    .2476457
------------------------------------------------------------------------------

. mixed attain  || schid:

Mixed-effects ML regression                     Number of obs     =      2,310
Group variable: schid                           Number of groups  =         17

                                                Obs per group:
                                                              min =         22
                                                              avg =      135.9
                                                              max =        286

                                                Wald chi2(0)      =          .
Log likelihood = -3221.0818                     Prob > chi2       =          .
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .0822691   .0756785     1.09   0.277    -.0660581    .2305963
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schid: Identity              |
                  var(_cons) |   .0887424   .0348585      .0410937    .1916405
-----------------------------+------------------------------------------------
               var(Residual) |   .9344115   .0276055      .8818425    .9901144
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 121.98        Prob >= chibar2 = 0.0000

. estat icc

Intraclass correlation
------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                       schid |   .0867342   .0312367      .0420038    .1706151
------------------------------------------------------------------------------
```

Let's compare the model where school effects are estimated as constrained random effects of dummy indicators with the one where neighborhood effects are estimated that way. Typically, we select which variable we would like to specify as clusters within _all depending on which one has fewer clusters because that will estimate faster. But also, if we want to have random slopes by cluster, we cannot specify that clustering variable within _all (we will see that below).

```
. mixed attain  || _all: R.schid || neighid:

Mixed-effects ML regression                     Number of obs     =      2,310

--------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+---------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
--------------------------------------------------------------

                                                Wald chi2(0)      =          .
```

```
Log likelihood = -3178.3557                     Prob > chi2       =          .
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .0753532   .0722216     1.04   0.297    -.0661987     .216905
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.schid) |    .075445   .0316491      .0331553    .1716755
-----------------------------+------------------------------------------------
neighid: Identity            |
                  var(_cons) |   .1412201   .0218651       .104257     .191288
-----------------------------+------------------------------------------------
               var(Residual) |   .7990182   .0263652      .7489788    .8524007
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 207.44            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. mixed attain  || _all: R.neighid || schid:

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
          schid |         17         22      135.9        286
-------------------------------------------------------------

                                                Wald chi2(0)      =          .
Log likelihood = -3178.3557                     Prob > chi2       =          .
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .0753532   .0722216     1.04   0.297    -.0661987     .216905
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
              var(R.neighid) |   .1412201   .0218651       .104257     .191288
-----------------------------+------------------------------------------------
schid: Identity              |
                  var(_cons) |    .075445   .0316491      .0331553    .1716755
-----------------------------+------------------------------------------------
               var(Residual) |   .7990182   .0263652      .7489788    .8524007
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 207.44            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Let’s add level 1 predictors to our model:

```
. mixed attain p7vrq p7read dadocc dadunemp daded momed male  || _all: R.schid ||
neighid:

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
```

```
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(7)      =    2415.24
Log likelihood = -2402.2937                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |     .02823   .0022758    12.40   0.000     .0237696    .0326904
      p7read |   .0269051   .0017587    15.30   0.000     .0234581    .0303521
      dadocc |   .0091773   .0013584     6.76   0.000     .0065148    .0118398
    dadunemp |  -.1464694      .0469    -3.12   0.002    -.2383916   -.0545471
       daded |   .1487033   .0410774     3.62   0.000      .068193    .2292136
       momed |   .0649316   .0376491     1.72   0.085    -.0088593    .1387225
        male |  -.0540241   .0285779    -1.89   0.059    -.1100358    .0019875
       _cons |   .0805349   .0272663     2.95   0.003      .027094    .1339758
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.schid) |   .0030572   .0024765      .0006249    .0149566
-----------------------------+------------------------------------------------
neighid: Identity            |
                  var(_cons) |   .0122113   .0072414      .0038194    .0390422
-----------------------------+------------------------------------------------
               var(Residual) |   .4551568   .0148446      .4269722    .4852018
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 7.40                  Prob > chi2 = 0.0248

Note: LR test is conservative and provided only for reference.
```

We can compare the variance components to the previous model; we see that much of school and neighborhood level variance has been explained. Let's explore random slopes for level 1 predictors:

```
. mixed attain p7vrq p7read dadocc dadunemp daded momed male  || _all: R.schid ||
neighid: p7vrq p7read dadocc dadunemp daded momed male

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(7)      =    2344.98
Log likelihood = -2400.0715                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |   .0282256   .0022713    12.43   0.000      .023774    .0326772
      p7read |   .0268055   .0017535    15.29   0.000     .0233688    .0302422
      dadocc |   .0089163   .0013768     6.48   0.000     .0062177    .0116149
    dadunemp |  -.1460657   .0476964    -3.06   0.002     -.239549   -.0525824
       daded |   .1524186   .0421274     3.62   0.000     .0698505    .2349868
       momed |   .0620998   .0393571     1.58   0.115    -.0150386    .1392382
```

```
        male |    -.05107   .0289833    -1.76   0.078    -.1078761    .0057362
       _cons |   .0773173   .0264105     2.93   0.003     .0255536     .129081
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
               var(R.schid) |   .0026377          .            .           .
-----------------------------+------------------------------------------------
neighid: Independent         |
                 var(p7vrq) |   1.26e-22          .            .           .
                var(p7read) |   2.00e-22          .            .           .
                var(dadocc) |   8.74e-06          .            .           .
              var(dadunemp) |   .0176157          .            .           .
                 var(daded) |   .0168711          .            .           .
                 var(momed) |   .0378511          .            .           .
                  var(male) |   .0115381          .            .           .
                 var(_cons) |   .0086013          .            .           .
-----------------------------+------------------------------------------------
              var(Residual) |   .4379232          .            .           .
------------------------------------------------------------------------------
LR test vs. linear model: chi2(9) = 11.84                 Prob > chi2 = 0.2224

Note: LR test is conservative and provided only for reference.

. mixed attain p7vrq p7read dadocc dadunemp daded momed male  || _all: R.schid  ||
neighid: dadocc dadunemp daded momed male

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(7)      =    2344.97
Log likelihood = -2400.0715                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |   .0282256   .0022713    12.43   0.000      .023774    .0326771
      p7read |   .0268055   .0017535    15.29   0.000     .0233688    .0302422
      dadocc |   .0089163   .0013769     6.48   0.000     .0062177    .0116149
    dadunemp |  -.1460655   .0476964    -3.06   0.002    -.2395488   -.0525823
       daded |   .1524182   .0421275     3.62   0.000     .0698499    .2349866
       momed |   .0621001   .0393571     1.58   0.115    -.0150384    .1392385
        male |  -.0510699   .0289832    -1.76   0.078    -.1078759    .0057362
       _cons |    .077317   .0264105     2.93   0.003     .0255533    .1290807
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
               var(R.schid) |   .0026377   .0023525      .0004592    .0151495
-----------------------------+------------------------------------------------
neighid: Independent         |
                var(dadocc) |   8.74e-06    .000039      1.41e-09     .054177
              var(dadunemp) |   .0176157   .0348477      .0003648    .8506678
                 var(daded) |   .0168731   .0286061      .0006083    .4680537
                 var(momed) |   .0378509     .02812      .0088248    .1623482
                  var(male) |   .0115365   .0158341       .000783     .169969
```

```
                 var(_cons) |    .008602    .007611      .0015186    .0487237
-----------------------------+------------------------------------------------
              var(Residual) |   .4379224   .0169907       .405856    .4725225
------------------------------------------------------------------------------
LR test vs. linear model: chi2(7) = 11.84                 Prob > chi2 = 0.1059

Note: LR test is conservative and provided only for reference.

. mixed attain p7vrq p7read dadocc dadunemp daded momed male  || _all: R.schid  ||
neighid: dadunemp daded momed male

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(7)      =    2352.90
Log likelihood = -2400.0975                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |    .028238   .0022708    12.44   0.000     .0237873    .0326887
      p7read |   .0267925   .0017535    15.28   0.000     .0233557    .0302294
      dadocc |   .0089268   .0013663     6.53   0.000      .006249    .0116047
    dadunemp |  -.1456732   .0477354    -3.05   0.002    -.2392329   -.0521135
       daded |   .1526091   .0421365     3.62   0.000     .0700231    .2351951
       momed |   .0619572   .0393694     1.57   0.116    -.0152055    .1391198
        male |  -.0513633   .0289873    -1.77   0.076    -.1081773    .0054507
       _cons |   .0777818   .0264007     2.95   0.003     .0260375    .1295261
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.schid) |   .0026311   .0023503      .0004569    .0151525
-----------------------------+------------------------------------------------
neighid: Independent         |
              var(dadunemp) |   .0181434   .0348393      .0004209    .7820113
                 var(daded) |   .0173458   .0284977       .000693    .4341538
                 var(momed) |   .0383208   .0280697      .0091187    .1610401
                  var(male) |   .0116597    .015826      .0008153    .1667422
                 var(_cons) |   .0087237   .0075905      .0015851    .0480106
-----------------------------+------------------------------------------------
              var(Residual) |   .4386847   .0166627      .4072124    .4725893
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 11.79                 Prob > chi2 = 0.0668

Note: LR test is conservative and provided only for reference.
```

We could do significance testing to see which slopes should be allowed to vary. We could also examine random slopes by school instead of neighborhoods – for that, we would reestimate it with _all: R.neigh. But for the sake of time, let's look at adding predictors on neighborhood and school level. Let's aggregate:

```
. for var p7vrq - male: bysort neighid: egen X_nei=mean(X)

->  bysort neighid: egen p7vrq_nei=mean(p7vrq)
```

```
->  bysort neighid: egen p7read_nei=mean(p7read)

->  bysort neighid: egen dadocc_nei=mean(dadocc)

->  bysort neighid: egen dadunemp_nei=mean(dadunemp)

->  bysort neighid: egen daded_nei=mean(daded)

->  bysort neighid: egen momed_nei=mean(momed)

->  bysort neighid: egen male_nei=mean(male)

. for var p7vrq - male: bysort schid: egen X_sch=mean(X)

->  bysort schid: egen p7vrq_sch=mean(p7vrq)

->  bysort schid: egen p7read_sch=mean(p7read)

->  bysort schid: egen dadocc_sch=mean(dadocc)

->  bysort schid: egen dadunemp_sch=mean(dadunemp)

->  bysort schid: egen daded_sch=mean(daded)

->  bysort schid: egen momed_sch=mean(momed)

->  bysort schid: egen male_sch=mean(male)

. mixed attain p7vrq p7read dadocc dadunemp daded momed male deprive p7vrq_nei
>  p7read_nei dadocc_nei dadunemp_nei daded_nei momed_nei male_nei  || _all: R
> .schid || neighid: dadunemp daded momed male

Mixed-effects ML regression                     Number of obs     =      2,310

-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(15)     =    2504.39
Log likelihood = -2373.3516                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |   .0260616   .0025478    10.23   0.000      .021068    .0310552
      p7read |   .0262333   .0020008    13.11   0.000     .0223118    .0301549
      dadocc |   .0060222   .0015824     3.81   0.000     .0029208    .0091236
    dadunemp |  -.1094969   .0535435    -2.05   0.041    -.2144402   -.0045537
       daded |   .1078427   .0468705     2.30   0.021     .0159782    .1997073
       momed |   .0585125   .0440004     1.33   0.184    -.0277268    .1447517
        male |   -.057422   .0320584    -1.79   0.073    -.1202553    .0054114
     deprive |  -.1068588   .0289249    -3.69   0.000    -.1635506   -.0501669
   p7vrq_nei |   .0076765   .0054024     1.42   0.155     -.002912    .0182651
  p7read_nei |  -.0022259   .0040418    -0.55   0.582    -.0101477    .0056959
  dadocc_nei |    .006009   .0031406     1.91   0.056    -.0001465    .0121644
dadunemp_nei |  -.0782511    .109447    -0.71   0.475    -.2927633    .1362611
   daded_nei |   .1801198   .0970098     1.86   0.063    -.0100158    .3702555
   momed_nei |  -.0423525   .0895377    -0.47   0.636    -.2178431    .1331381
    male_nei |   .0428611   .0705518     0.61   0.544    -.0954179      .18114
       _cons |   .0505329   .0439281     1.15   0.250    -.0355646    .1366304
```

```
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
               var(R.schid) |   .0037432   .0025493      .0009852    .0142217
-----------------------------+------------------------------------------------
neighid: Independent         |
              var(dadunemp) |   .0073473   .0328626      1.15e-06    47.13606
                 var(daded) |   .0115766   .0271607      .0001165    1.149878
                 var(momed) |   .0352373   .0264951       .008072    .1538242
                  var(male) |   .0103147   .0142086      .0006933    .1534618
                 var(_cons) |   8.08e-12   1.81e-11      9.89e-14    6.60e-10
-----------------------------+------------------------------------------------
              var(Residual) |   .4387169   .0159951      .4084609    .4712139
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 10.55                 Prob > chi2 = 0.1034

Note: LR test is conservative and provided only for reference.

. mixed attain p7vrq p7read dadocc dadunemp daded momed male deprive p7vrq_sch
p7read_sch dadocc_sch dadunemp_sch daded_sch momed_sch male_sch  || _all: R.schid ||
neighid: dadunemp daded momed male

Mixed-effects ML regression                     Number of obs     =      2,310
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      2,310    2,310.0      2,310
        neighid |        524          1        4.4         16
-------------------------------------------------------------

                                                Wald chi2(15)     =    2554.09
Log likelihood = -2378.1708                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
      attain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       p7vrq |   .0273469    .002274    12.03   0.000     .0228899    .0318038
      p7read |   .0262508   .0017552    14.96   0.000     .0228108    .0296909
      dadocc |   .0077382   .0013713     5.64   0.000     .0050504    .0104259
    dadunemp |  -.1256167   .0477079    -2.63   0.008    -.2191225   -.0321109
       daded |   .1495834   .0420635     3.56   0.000     .0671404    .2320264
       momed |   .0545836   .0391762     1.39   0.164    -.0222003    .1313675
        male |  -.0544371   .0288055    -1.89   0.059    -.1108948    .0020207
     deprive |  -.1631975   .0262269    -6.22   0.000    -.2146014   -.1117937
   p7vrq_sch |   .0263455   .0171515     1.54   0.125    -.0072708    .0599618
  p7read_sch |  -.0199684   .0135388    -1.47   0.140    -.0465039    .0065672
  dadocc_sch |   .0248499   .0191866     1.30   0.195    -.0127552     .062455
dadunemp_sch |   1.113901   .5183483     2.15   0.032     .0979574    2.129846
   daded_sch |  -.6698207    .447016    -1.50   0.134    -1.545956    .2063146
   momed_sch |   .4964818   .6049023     0.82   0.412    -.6891049    1.682069
    male_sch |   .3916443    .416703     0.94   0.347    -.4250787    1.208367
       _cons |   -.210726   .2503211    -0.84   0.400    -.7013464    .2798944
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
               var(R.schid) |   .0006947   .0015285      9.31e-06    .0518265
-----------------------------+------------------------------------------------
neighid: Independent         |
              var(dadunemp) |   .0178082     .03453      .0003982    .7963269
```

```
                 var(daded) |   .0185407   .0282593      .0009348     .367715
                 var(momed) |   .0377594   .0272576      .0091741    .1554129
                  var(male) |   .0100372   .0143958      .0006037    .1668946
                 var(_cons) |   1.00e-14   2.19e-14      1.37e-16    7.33e-13
-----------------------------+------------------------------------------------
              var(Residual) |   .4395822   .0159506      .4094055    .4719832
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 5.69                 Prob > chi2 = 0.4594

Note: LR test is conservative and provided only for reference.
```

We should probably mean-center some predictors, and also examine which of them should stay (most are not significant). We could also examine some cross-level interactions, but in general, there is not that much slope variance, and very little school-level and neighborhood-level unexplained variance left.

Cross-nested models for longitudinal data

Cross-nested models can also be used with longitudinal data like NYS dataset we used earlier. Here, we can treat observations as nested both within individuals and within time points. That is, in our earlier analysis, we assumed that the trajectory of age (from 14 to 18) is linear and student-specific (a random slope). In this cross-nested model, we will assume that the effect due to specific age is systematic to that age and common to all students. The rationale behind that could be assuming that the students were measured contemporaneously, and there may have been some year-specific factors happening that affected all the students, making everyone's deviance attitude score either higher or lower in that given year. So we will estimate a two-way crossed-nested model, with the student effects u_i being crossed with the age effects v_j:

```
. use nys.dta, clear

. reshape long attit expo, i(id) j(age)
(note: j = 14 15 16 17 18)

Data                               wide   ->   long
-----------------------------------------------------------------------------
Number of obs.                      241   ->    1205
Number of variables                  14   ->       7
j variable (5 values)                     ->   age
xij variables:
            attit14 attit15 ... attit18   ->   attit
               expo14 expo15 ... expo18   ->   expo
-----------------------------------------------------------------------------

. mixed attit || _all: R.age || id:

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood =  34.695287
Iteration 1:   log likelihood =  34.695287

Computing standard errors:

Mixed-effects ML regression                     Number of obs     =      1,066

-------------------------------------------------------------
```

```
                 |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      1,066    1,066.0      1,066
             id |        241          1        4.4          5
-------------------------------------------------------------

                                                Wald chi2(0)      =          .
Log likelihood =  34.695287                     Prob > chi2       =          .

------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .4923777   .0267443    18.41   0.000     .4399598    .5447955
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                  var(R.age) |    .002475   .0016883        .00065    .0094237
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   .0443494   .0048906       .035729    .0550496
-----------------------------+------------------------------------------------
               var(Residual) |   .0357432   .0017678      .0324411    .0393815
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 415.72                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

For this kind of model to make sense, we should have more time points that we do here – 5 time points is better for fixed effects of time rather than for a random variable – we do it here for demonstration purposes only. So this kind of analysis makes sense if you have a LONG time-series rather than a short one.

We can now add the trajectory, first linear non-varying, then linear randomly varying, then quadratic randomly varying:

```
. gen age16=age-16

. mixed attit age16  || _all: R.age16 || id:

Mixed-effects ML regression                     Number of obs     =      1,066
-------------------------------------------------------------
                 |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      1,066    1,066.0      1,066
             id |        241          1        4.4          5
-------------------------------------------------------------

                                                Wald chi2(1)      =      20.21
Log likelihood =  38.657761                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0322932   .0071842     4.50   0.000     .0182124     .046374
       _cons |    .492564   .0169588    29.04   0.000     .4593255    .5258026
------------------------------------------------------------------------------
------------------------------------------------------------------------------
```

```
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.age16) |   .0003369   .0003345      .0000481    .0023583
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   .0443475   .0048856      .0357351    .0550355
-----------------------------+------------------------------------------------
               var(Residual) |   .0357263   .0017662      .0324271    .0393611
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 401.35                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.


. mixed attit age16  || _all: R.age16 || id: age16

Mixed-effects ML regression                     Number of obs     =      1,066
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      1,066    1,066.0      1,066
             id |        241          1        4.4          5
-------------------------------------------------------------

                                                Wald chi2(1)      =      15.60
Log likelihood =  60.274204                     Prob > chi2       =     0.0001
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |    .032264   .0081687     3.95   0.000     .0162537    .0482743
       _cons |   .4929646   .0172185    28.63   0.000     .4592169    .5267122
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.age16) |   .0003816    .000345      .0000649    .0022449
-----------------------------+------------------------------------------------
id: Independent              |
                  var(age16) |   .0031574   .0006363      .0021271    .0046868
                  var(_cons) |   .0455911   .0048767      .0369683    .0562251
-----------------------------+------------------------------------------------
               var(Residual) |   .0282275   .0016269      .0252124    .0316033
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 444.59                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. mixed attit c.age16##c.age16  || _all: R.age16 || id: c.age16##c.age16

Mixed-effects ML regression                     Number of obs     =      1,066
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      1,066    1,066.0      1,066
             id |        241          1        4.4          5
-------------------------------------------------------------

                                                Wald chi2(2)      =      38.20
Log likelihood =  65.811166                     Prob > chi2       =     0.0000
```

```
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0319722   .0057991     5.51   0.000     .0206062    .0433381
             |
     c.age16#|
     c.age16 |   -.010505   .0038782    -2.71   0.007    -.0181062   -.0029038
             |
       _cons |   .5137537   .0167022    30.76   0.000      .481018    .5464895
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.age16) |   .0000505    .000123      4.25e-07    .0059915
-----------------------------+------------------------------------------------
id: Independent              |
                  var(age16) |   .0034223   .0006344      .0023797    .0049217
            var(age16#age16) |   .0005715   .0002493      .0002431    .0013437
                  var(_cons) |   .0466065   .0051029      .0376053    .0577622
-----------------------------+------------------------------------------------
               var(Residual) |   .0251662   .0018311      .0218214    .0290237
------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 450.29                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Here we note that the variance for the random effects of time points is very close to 0 – it doesn't seem that individual time points have much of an effect beyond what's specified by random trajectory. Let's compare some specifications of time here:

```
. est store quadratic_re

. mixed attit c.age16##c.age16 || id: c.age16##c.age16

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(2)      =      46.07
Log likelihood =  65.681561                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0319353   .0053468     5.97   0.000     .0214557    .0424148
             |
     c.age16#|
     c.age16 |  -.0105168   .0033811    -3.11   0.002    -.0171436     -.00389
             |
       _cons |   .5137654   .0159545    32.20   0.000     .4824951    .5450357
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Independent              |
                  var(age16) |   .0034152    .000635      .0023721    .0049169
            var(age16#age16) |   .0005646   .0002491      .0002378    .0013407
```

```
                 var(_cons) |   .0465887   .0051017      .0375897     .057742
-----------------------------+------------------------------------------------
              var(Residual) |   .0252557   .0018322      .0219083    .0291146
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 450.03                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. est store quadratic

. est stats quadratic quadratic_re

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
   quadratic |      1,066          .   65.68156       7  -117.3631  -82.56144
quadratic_re |      1,066          .   65.81117       8  -115.6223  -75.84898
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. lrtest quadratic quadratic_re

Likelihood-ratio test                                 LR chi2(1)  =      0.26
(Assumption: quadratic nested in quadratic_re)        Prob > chi2 =    0.6107

Note: The reported degrees of freedom assumes the null hypothesis is not on
      the boundary of the parameter space.  If this is not true, then the
      reported test is conservative.

. mixed attit c.age16##c.age16  || _all: R.age16 || id:

Mixed-effects ML regression                     Number of obs     =      1,066
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1      1,066    1,066.0      1,066
             id |        241          1        4.4          5
-------------------------------------------------------------

                                                Wald chi2(2)      =      67.13
Log likelihood =  41.042581                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0322139   .0042397     7.60   0.000     .0239042    .0405236
             |
     c.age16#|
     c.age16 |  -.0103426   .0034938    -2.96   0.003    -.0171903   -.0034948
             |
       _cons |   .5130924   .0163665    31.35   0.000     .4810147    .5451701
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                var(R.age16) |   4.80e-07   .0001233     8.4e-226    2.7e+212
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |    .044296   .0048806     .0356925    .0549734
-----------------------------+------------------------------------------------
```

```
              var(Residual) |   .0357288   .0017669      .0324282    .0393653
-------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 400.75                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. est store noslopevar

. est stats quadratic quadratic_re noslopevar

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
   quadratic |      1,066          .   65.68156       7  -117.3631  -82.56144
quadratic_re |      1,066          .   65.81117       8  -115.6223  -75.84898
  noslopevar |      1,066          .   41.04258       6  -70.08516  -40.25515
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.
```

The model with just the randomly varying quadratic trajectory seems to be better than either the model with the randomly varying quadratic trajectory PLUS individual time point effects, or the model with just the individual time point effects and non-varying quadratic trajectory. So we would probably set aside the idea of cross-nested model here and stick with the randomly varying quadratic trajectory model. But otherwise, we could continue adding predictors etc.