Longitudinal Data Analysis
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Panel Data Analysis: Mixed Effects Models, Part 2

<u>Using margins and marginsplot to interpret interactions in mixed effects models</u>

We can use the following set of tools to generate predicted slopes and graphs to assist interpretation of mixed effects models; note that these interpretation tools can be used to interpret the results of other types of regression models as well (OLS, FE, RE, etc.).

Interactions of expochange, income, and minority

When starting interpretation, we have to select one variable as a focal variable and the other as a moderator. It usually makes more sense to select level 1 variable as focal, so we will use expochange as focal and income and minority as moderators. First, we focus on income. To evaluate the size of the effect of expochange at different levels of income, we can use margins command. For that, we reestimate the model with interactions syntax using the original income (non-mean centered, to be able to specify categories in integers).

```
. mixed attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.expochange##i.minority c.expochange##c.income || id: c.age16##c.age16 expochange
note: age16 omitted because of collinearity
note: expochange omitted because of collinearity

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(13)     =     418.22
Log likelihood =   235.0784                     Prob > chi2       =     0.0000
-----------------------------------------------------------------------------------
                    attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------------+--------------------------------------------------------
                    age16 |   .0219327      .00646     3.40   0.001     .0092713    .0345941
                          |
           c.age16#c.age16 |    -.01207    .0039917    -3.02   0.002    -.0198936   -.0042463
                          |
                expomeanm |   .6327828    .0462137    13.69   0.000     .5422056      .72336
                          |
       c.age16#c.expomeanm |  -.0425937    .0186781    -2.28   0.023     -.079202   -.0059853
                          |
c.age16#c.age16#c.expomeanm |  -.0085125    .0118275    -0.72   0.472    -.0316939    .0146689
                          |
                    age16 |          0  (omitted)
                 1.female |  -.0740172    .0238309    -3.11   0.002     -.120725   -.0273095
                          |
            female#c.age16 |
                        1  |   .0037775    .0095619     0.40   0.693    -.0149635    .0225186
                          |
    female#c.age16#c.age16 |
                        1  |   .0151617    .0059456     2.55   0.011     .0035085    .0268148
                          |
               expochange |   .6396815    .0857774     7.46   0.000     .4715608    .8078022
```

```
             1.minority |   .0008616   .0270481     0.03   0.975    -.0521516    .0538749
                        |
  minority#c.expochange |
                     1  |  -.3068606   .0890185    -3.45   0.001    -.4813336   -.1323877
                        |
             expochange |          0  (omitted)
                 income |   .0140268   .0047571     2.95   0.003      .004703    .0233506
                        |
  c.expochange#c.income |  -.0535635   .0164859    -3.25   0.001    -.0858753   -.0212517
                        |
                  _cons |   .4770449   .0277167    17.21   0.000     .4227212    .5313686
-------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Independent              |
                  var(age16) |   .0023462   .0005053      .0015383    .0035784
            var(age16#age16) |   .0002081   .0001693      .0000422    .0010254
             var(expochange) |   .0553525   .0207923      .0265092    .1155786
                  var(_cons) |   .0204894   .0024236      .0162497    .0258354
-----------------------------+------------------------------------------------
               var(Residual) |   .0205676   .0014936       .017839    .0237137
------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 266.69                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Since expochange also interacts with minority, we should decide at which value of minority variable we'd like to estimate the coefficient of exposure – let's say, we focus on non-minorities for now. Expochange is specified in dydx option, and we evaluate it at income ranging from 1 to 10 for non-minorities:

```
. margins, dydx(expochange) at(income=(1(1)10) minority=0)

Average marginal effects                        Number of obs     =      1,066

Expression   : Linear prediction, fixed portion, predict()
dy/dx w.r.t. : expochange

1._at        : minority        =           0
               income          =           1

2._at        : minority        =           0
               income          =           2

3._at        : minority        =           0
               income          =           3

4._at        : minority        =           0
               income          =           4

5._at        : minority        =           0
               income          =           5

6._at        : minority        =           0
               income          =           6

7._at        : minority        =           0
               income          =           7

8._at        : minority        =           0
               income          =           8
```

```
9._at        : minority        =           0
               income          =           9

10._at       : minority        =           0
               income          =          10

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
expochange   |
         _at |
          1  |    .586118    .071863     8.16   0.000     .4452691    .7269668
          2  |   .5325545   .0592822     8.98   0.000     .4163635    .6487454
          3  |   .4789909   .0490717     9.76   0.000     .3828121    .5751697
          4  |   .4254274   .0429565     9.90   0.000     .3412342    .5096206
          5  |   .3718639   .0427325     8.70   0.000     .2881098    .4556179
          6  |   .3183003   .0484813     6.57   0.000     .2232788    .4133219
          7  |   .2647368   .0584669     4.53   0.000     .1501439    .3793297
          8  |   .2111733   .0709217     2.98   0.003     .0721693    .3501772
          9  |   .1576097   .0847642     1.86   0.063     -.008525    .3237445
         10  |   .1040462   .0994164     1.05   0.295    -.0908063    .2988988
------------------------------------------------------------------------------
```

So here we see that above non-minorities, exposure has a significant positive effect on attitudes for income ranging from 1 to 8, but in categories 9 and 10, it does not. We could also check for minorities if we wanted.

What if we want to know the effect of exposure when income is one standard deviation above/below the mean? We can find 1 SD, and then use margins at those values:

```
. sum income

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
      income |      1,066     4.15197    2.400583          1         10

. di r(mean)+r(sd)
6.552553

. di r(mean)-r(sd)
1.7513869

. margins, dydx(expochange) at(income=(1.7513869 4.15197 6.552553)) atmeans

Conditional marginal effects                    Number of obs     =      1,066

Expression   : Linear prediction, fixed portion, predict()
dy/dx w.r.t. : expochange

1._at        : age16           =   -.0262664 (mean)
               expomeanm       =   -3.65e-10 (mean)
               0.female        =    .5562852 (mean)
               1.female        =    .4437148 (mean)
               expochange      =   -1.47e-09 (mean)
               0.minority      =    .7786116 (mean)
               1.minority      =    .2213884 (mean)
               income          =    1.751387

2._at        : age16           =   -.0262664 (mean)
```

```
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =   -1.47e-09 (mean)
                 0.minority      =    .7786116 (mean)
                 1.minority      =    .2213884 (mean)
                 income          =     4.15197

3._at          : age16           =   -.0262664 (mean)
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =   -1.47e-09 (mean)
                 0.minority      =    .7786116 (mean)
                 1.minority      =    .2213884 (mean)
                 income          =    6.552553

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
expochange   |
         _at |
          1  |   .4779357   .0524154     9.12   0.000     .3752035    .5806679
          2  |    .349352   .0363602     9.61   0.000     .2780873    .4206167
          3  |   .2207683   .0550386     4.01   0.000     .1128946    .3286419
------------------------------------------------------------------------------
```

Here, when income is 1 SD above the mean, 1 unit increase in exposure over time translates into .22 increase in attitudes; when income is 1SD below the mean, 1 unit increase in exposure over time translates into .48 increase in attitudes. This is controlling for minorities & non-minorities combined; we could further separate by adding minority to the "at" statement of margins.

Now let's do some graphical examination. Generating predictions at a few levels of income, minority, and expochange to be used in graphs; everything else held at means.

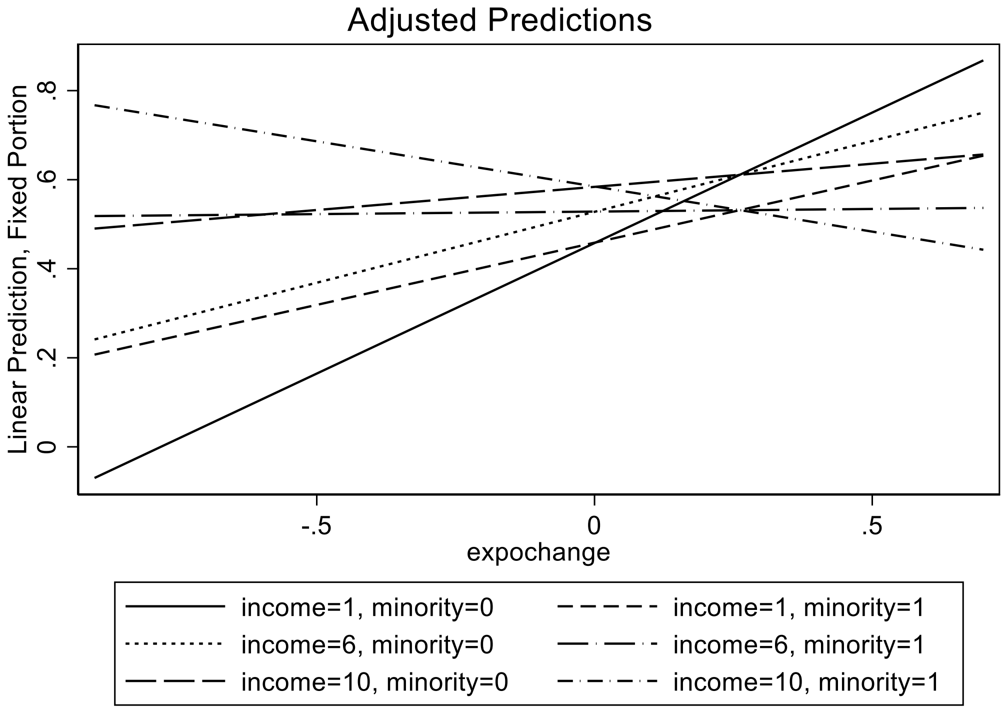
```
. margins, at(income=(1, 6, 10) minority=(0/1) expochange=(-.9, .7)) atmeans

Adjusted predictions                              Number of obs     =      1,066

Expression     : Linear prediction, fixed portion, predict()

1._at          : age16           =   -.0262664 (mean)
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =         -.9
                 minority        =           0
                 income          =           1

2._at          : age16           =   -.0262664 (mean)
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =         -.9
                 minority        =           0
                 income          =           6

3._at          : age16           =   -.0262664 (mean)
                 expomeanm       =   -3.65e-10 (mean)
```

```
                          0.female        =    .5562852 (mean)
                          1.female        =    .4437148 (mean)
                          expochange      =         -.9
                          minority        =           0
                          income          =          10

4._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =         -.9
                  minority        =           1
                  income          =           1

5._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =         -.9
                  minority        =           1
                  income          =           6

6._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =         -.9
                  minority        =           1
                  income          =          10

7._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =          .7
                  minority        =           0
                  income          =           1

8._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =          .7
                  minority        =           0
                  income          =           6

9._at           : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =          .7
                  minority        =           0
                  income          =          10

10._at          : age16           =   -.0262664 (mean)
                  expomeanm       =   -3.65e-10 (mean)
                  0.female        =    .5562852 (mean)
                  1.female        =    .4437148 (mean)
                  expochange      =          .7
                  minority        =           1
                  income          =           1

11._at          : age16           =   -.0262664 (mean)
```

```
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =          .7
                 minority        =           1
                 income          =           6

12._at       : age16           =   -.0262664 (mean)
                 expomeanm       =   -3.65e-10 (mean)
                 0.female        =    .5562852 (mean)
                 1.female        =    .4437148 (mean)
                 expochange      =          .7
                 minority        =           1
                 income          =          10

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |  -.0699008   .0687811    -1.02   0.309    -.2047093    .0649077
          2  |   .2412692   .0465762     5.18   0.000     .1499814    .3325569
          3  |   .4902052   .0941735     5.21   0.000     .3056285    .6747818
          4  |   .2071354   .0701499     2.95   0.003      .069644    .3446268
          5  |   .5183054   .0875903     5.92   0.000     .3466316    .6899791
          6  |   .7672414   .1364602     5.62   0.000     .4997843    1.034698
          7  |    .867888   .0540775    16.05   0.000     .7618981    .9738778
          8  |   .7505497   .0364273    20.60   0.000     .6791535    .8219459
          9  |   .6566791   .0751755     8.74   0.000     .5093379    .8040203
         10  |   .6539472   .0563812    11.60   0.000     .5434421    .7644523
         11  |   .5366089   .0707578     7.58   0.000     .3979262    .6752917
         12  |   .4427383   .1098693     4.03   0.000     .2273984    .6580783
------------------------------------------------------------------------------
```

The command for graphs, marginsplot, which plots the results of margins immediately preceding it. Note that I focus on expochange for X axis, specify noci to suppress confidence intervals, label x axis with three points of interest, and suppress dots with msymbol i – I want just the lines:

```
. marginsplot, x(expochange) noci xlabel(-.5 0 .5) plotop(msymbol(i))

  Variables that uniquely identify margins: income minority expochange
```

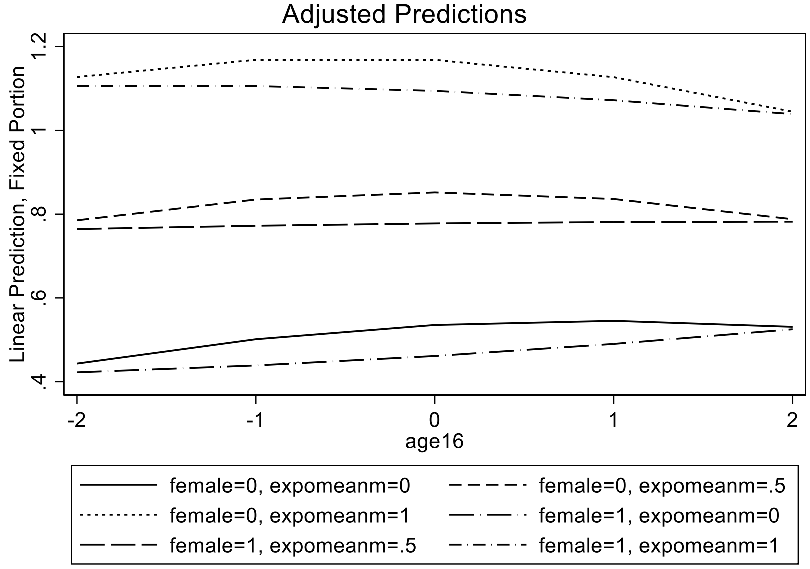


We could further play with color scheme, legend etc., but this is clear enough.

Age, gender, and average exposure interactions

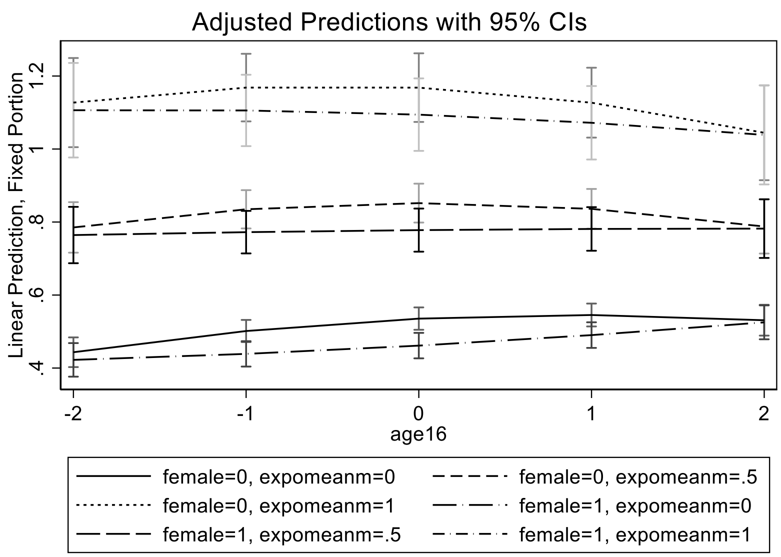
Now let's explore the squared term of age. Age interacts with gender and average exposure. The values for gender are clear, 0/1, but what values of expomean to use for our graph?

```
. sum expomean

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
    expomean |      1,066    .5601501    .2532099          0       1.32

.  qui margins, at(age16=(-2(1)2) female=(0/1) expomean=(0(.5)1)) atmeans

. marginsplot, x(age16) noci plotop(msymbol(i)) scheme(s1mono)

  Variables that uniquely identify margins: age16 female expomean
```



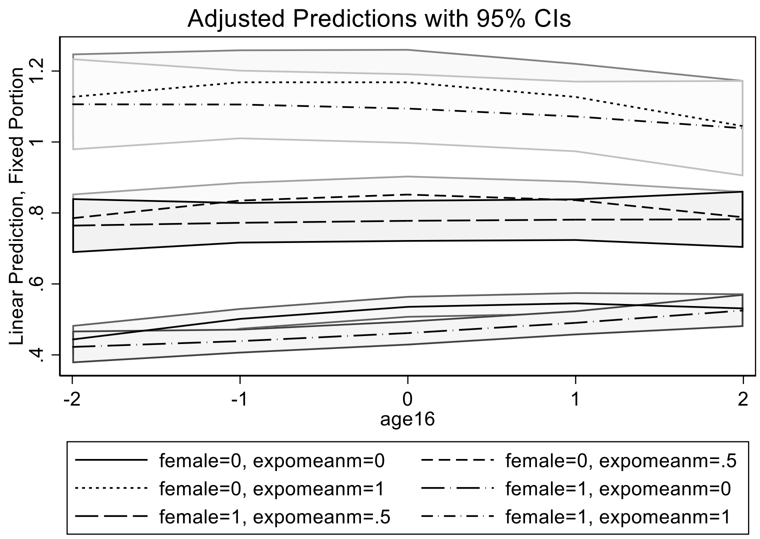
Now with confidence intervals:

```
. qui margins, at(age16=(-2(1)2) female=(0/1) expomean=(0(.5)1)) atmeans

. marginsplot, x(age16) plotop(msymbol(i)) scheme(s1mono)

  Variables that uniquely identify margins: age16 female expomean
```



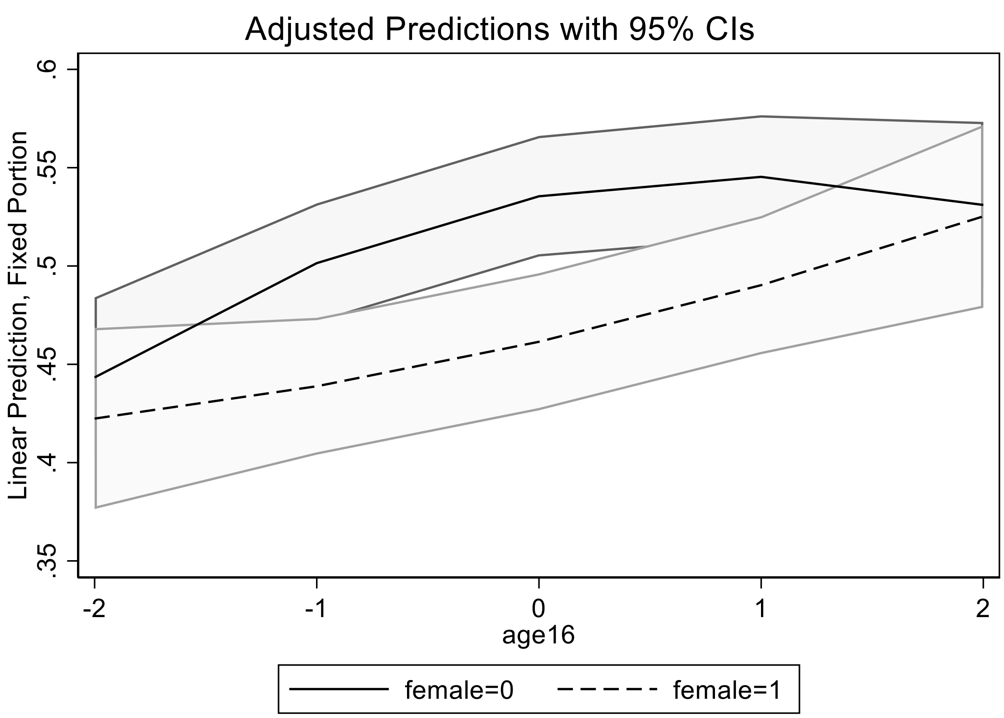
Or better yet as shaded areas:

```
. marginsplot, x(age16) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
scheme(s1mono)

  Variables that uniquely identify margins: age16 female expomean
```



Works better with fewer groups, for example, focusing on gender and keeping exposure at its mean:

```
. qui margins, at(age16=(-2(1)2) female=(0/1)) atmeans

. marginsplot, x(age16) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
scheme(s1mono)

  Variables that uniquely identify margins: age16 female
```



Calculating percentage variance explained

Next, let's explore how much variance in slopes of each of level 1 predictors our final model explain. For that, let's compare it to a model without any level 2 predictors:

```
. mixed attit c.age16##c.age16  expochange  || id: c.age16##c.age16 expochange,
cov(unstructured)

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(3)      =     136.40
Log likelihood =  143.93827                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
          attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------+---------------------------------------------------------------
          age16 |   .0244353   .0049082     4.98   0.000     .0148153    .0340552
                |
c.age16#c.age16 |  -.0055659   .0032361    -1.72   0.085    -.0119085    .0007766
                |
     expochange |    .340693   .0374501     9.10   0.000     .2672923    .4140938
          _cons |   .5031676   .0164084    30.67   0.000     .4710078    .5353274
--------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0027061   .0005383      .0018324    .0039963
            var(age16#age16) |   .0006854   .0002464      .0003388    .0013868
             var(expochange) |   .0723975   .0239199      .0378869    .1383433
                  var(_cons) |   .0527191   .0059834      .0422046    .0658531
       cov(age16,age16#age16) |  -.0002788   .0002435     -.0007562    .0001985
        cov(age16,expochange) |  -.0029004   .0026057     -.0080074    .0022066
             cov(age16,_cons) |  -.0009484   .0012398     -.0033784    .0014816
 cov(age16#age16,expochange) |   .0001815   .0015366     -.0028302    .0031933
      cov(age16#age16,_cons) |   -.002066   .0009304     -.0038897   -.0002424
        cov(expochange,_cons) |    .014326   .0095988     -.0044873    .0331392
-----------------------------+------------------------------------------------
               var(Residual) |   .0190377   .0014571      .0163857     .022119
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 546.98               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov

Random-effects covariance matrix for level id

             |     age16  c.c_ag~16  expocha~e      _cons
-------------+--------------------------------------------
       age16 |  .0027061
c.c_age16~16 | -.0002788   .0006854
  expochange | -.0029004   .0001815   .0723975
       _cons | -.0009484   -.002066    .014326   .0527191
```

```
. mixed attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.expochange##i.minority c.expochange##c.incomem  || id: c.age16##c.age16 expochange,
cov(unstructured)
note: age16 omitted because of collinearity
note: expochange omitted because of collinearity

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(13)     =     430.09
Log likelihood =  240.81527                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------------
                     attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------------+--------------------------------------------------------------------
                     age16 |   .0219426   .0064814     3.39   0.001     .0092393    .0346459
                           |
           c.age16#c.age16 |  -.0122447   .0042228    -2.90   0.004    -.0205212   -.0039681
                           |
                 expomeanm |   .6318958    .049011    12.89   0.000      .535836    .7279556
                           |
       c.age16#c.expomeanm |  -.0408274   .0187166    -2.18   0.029    -.0775112   -.0041436
                           |
c.age16#c.age16#c.expomeanm |  -.0091057   .0124199    -0.73   0.463    -.0334482    .0152368
                           |
                     age16 |          0  (omitted)
                  1.female |  -.0738328   .0252351    -2.93   0.003    -.1232926    -.024373
                           |
            female#c.age16 |
                        1  |   .0039725   .0095473     0.42   0.677    -.0147399    .0226849
                           |
    female#c.age16#c.age16 |
                        1  |   .0151555   .0062652     2.42   0.016     .0028759    .0274351
                           |
                expochange |   .4158301   .0420899     9.88   0.000     .3333353    .4983248
                1.minority |   .0038862   .0267865     0.15   0.885    -.0486143    .0563868
                           |
     minority#c.expochange |
                        1  |  -.3121521   .0882582    -3.54   0.000    -.4851349   -.1391693
                           |
                expochange |          0  (omitted)
                   incomem |   .0152012   .0047064     3.23   0.001     .0059768    .0244256
                           |
    c.expochange#c.incomem |  -.0550901    .016154    -3.41   0.001    -.0867514   -.0234288
                           |
                     _cons |   .5347902   .0180973    29.55   0.000     .4993201    .5702604
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0026167   .0005263      .0017642     .003881
            var(age16#age16) |   .0005755   .0002339      .0002595    .0012763
             var(expochange) |   .0574034   .0207247      .0282894    .1164801
                  var(_cons) |    .024968   .0033839      .0191435    .0325648
      cov(age16,age16#age16) |  -.0003702   .0002346       -.00083    .0000897
        cov(age16,expochange) |  -.0030557   .0024417     -.0078412    .0017299
             cov(age16,_cons) |   .0013106   .0009329     -.0005178     .003139
 cov(age16#age16,expochange) |   .0015322   .0014623     -.0013339    .0043982
      cov(age16#age16,_cons) |  -.0015776   .0007069     -.0029631    -.000192
        cov(expochange,_cons) |   .0017044   .0066061     -.0112434    .0146521
-----------------------------+------------------------------------------------
```

```
               var(Residual) |   .0191298   .0014506      .0164878     .022195
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 278.16               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov

Random-effects covariance matrix for level id

             |     age16  c.c_ag~16  expocha~e      _cons
-------------+--------------------------------------------
       age16 |  .0026167
c.c_age16~16 | -.0003702   .0005755
  expochange | -.0030557   .0015322   .0574034
       _cons |  .0013106  -.0015776   .0017044    .024968
```

To see both matrices back to back, I copied the one from the model above:

```
             |     age16  c.c_ag~16  expocha~e      _cons
-------------+--------------------------------------------
       age16 |  .0027061
c.c_age16~16 | -.0002788   .0006854
  expochange | -.0029004   .0001815   .0723975
       _cons | -.0009484   -.002066    .014326   .0527191
```

So the variance in age16 slope went from .0027061 to .0026167, variance in age16 squared slope from .0006854 to .0005755, variance in expochange slope from .0723975 to .0574034, and variance in intercepts from .0527191 to .024968. We can calculate all of these as percentages:

```
. di (.0027061-.0026167)/.0027061
.03303647

. di (.0006854-.0005755)/.0006854
.16034432

. di (.0723975-.0574034)/.0723975
.20710798

. di (.0527191-.024968)/.0527191
.52639556
```

We might also want to know overall how much of within-person and between-person variance we are explaining here. Typically, in growth curve models, we compare our full model to a model with only time variables (in our case, age and age squared) that does not include any random slopes, only random intercept. Here’s our null model:

```
. mixed attit c.age16##c.age16 || id:

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood =  41.042571
Iteration 1:   log likelihood =  41.042571

Computing standard errors:

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241
```

```
                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(2)      =      67.32
Log likelihood =  41.042571                     Prob > chi2       =     0.0000

--------------------------------------------------------------------------------
          attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------+---------------------------------------------------------------
          age16 |   .0322138   .0042341     7.61   0.000     .0239152    .0405125
                |
c.age16#c.age16 |  -.0103428   .0034889    -2.96   0.003    -.0171809   -.0035046
                |
          _cons |   .5130925   .0163594    31.36   0.000     .4810287    .5451563
--------------------------------------------------------------------------------

-----------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+-----------------------------------------------
id: Identity                 |
                  var(_cons) |    .044296   .0048807      .0356925    .0549734
-----------------------------+-----------------------------------------------
               var(Residual) |   .0357293   .0017625      .0324366    .0393562
-----------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 400.75        Prob >= chibar2 = 0.0000

. mixed attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.expochange##i.minority c.expochange##c.incomem
> || id:
note: age16 omitted because of collinearity
note: expochange omitted because of collinearity

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood =  210.19859
Iteration 1:   log likelihood =  210.19859

Computing standard errors:

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(13)     =     512.91
Log likelihood =  210.19859                     Prob > chi2       =     0.0000

-----------------------------------------------------------------------------------------
                    attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------------+--------------------------------------------------------------
                    age16 |   .0204332    .005275     3.87   0.000     .0100943     .030772
                          |
          c.age16#c.age16 |  -.0116962   .0042534    -2.75   0.006    -.0200326   -.0033598
                          |
                expomeanm |   .6296548   .0482303    13.06   0.000     .5351253    .7241844
                          |
      c.age16#c.expomeanm |  -.0426656    .015232    -2.80   0.005    -.0725199   -.0128114
```

```
                           |
c.age16#c.age16#c.expomeanm |  -.0071113   .0125844    -0.57   0.572    -.0317763    .0175536
                           |
                     age16 |          0  (omitted)
                  1.female |  -.0723951   .0248503    -2.91   0.004    -.1211009   -.0236893
                           |
            female#c.age16 |
                        1  |   .0039387   .0077084     0.51   0.609    -.0111694    .0190469
                           |
    female#c.age16#c.age16 |
                        1  |   .0148067   .0063583     2.33   0.020     .0023447    .0272687
                           |
                expochange |   .4421253   .0357469    12.37   0.000     .3720626     .512188
                1.minority |   .0023037    .027007     0.09   0.932    -.0506291    .0552364
                           |
     minority#c.expochange |
                        1  |  -.3099774   .0737696    -4.20   0.000    -.4545631   -.1653917
                           |
                expochange |          0  (omitted)
                   incomem |    .014602   .0047458     3.08   0.002     .0053005    .0239036
                           |
    c.expochange#c.incomem |   -.047813   .0140664    -3.40   0.001    -.0753827   -.0202434
                           |
                     _cons |   .5334658   .0178822    29.83   0.000     .4984174    .5685142
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   .0194187   .0024217      .0152078    .0247955
-----------------------------+------------------------------------------------
               var(Residual) |   .0290408   .0014289      .0263709    .0319809
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 216.93        Prob >= chibar2 = 0.0000
```

So for level 1, we have var(Residual) .0290408 vs .0357293 and for level 2, var(_cons) is .019418 vs .044296. Now we can calculate R squared:

```
* Level 1:
. di (.0357293 -.0290408)/.0357293
.1871993
* Level 2:
. di (.044296-.019418)/.044296
.56163085
* Total R-squared:
. di (.0357293 - .0290408 + .044296-.019418)/(.044296+.0357293)
.3944565
```

We can also run xtreg to obtain approximate R-squared values (since we do not need random slopes for that) but those R squared values will compare our model to a null model without any variables (so without age variables as well), so they will be somewhat different:

```
. xtreg attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.expochange##i.minority c.expochange##c.incomem,
> re
note: age16 omitted because of collinearity
note: expochange omitted because of collinearity

Random-effects GLS regression                   Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

R-sq:                                           Obs per group:
     within  = 0.2477                                         min =          1
```

```
     between = 0.5058                                         avg =        4.4
     overall = 0.4028                                         max =          5

                                                Wald chi2(13)     =     504.52
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------------------
                     attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------------+--------------------------------------------------------------
                     age16 |   .0204333   .0053028     3.85   0.000     .0100401    .0308266
                           |
          c.age16#c.age16  |  -.0116914   .0042752    -2.73   0.006    -.0200706   -.0033122
                           |
                 expomeanm |   .6297262   .0487234    12.92   0.000     .5342301    .7252222
                           |
       c.age16#c.expomeanm |  -.0426375   .0153122    -2.78   0.005    -.0726489   -.0126261
                           |
c.age16#c.age16#c.expomeanm |  -.0071247   .0126492    -0.56   0.573    -.0319167    .0176673
                           |
                     age16 |          0  (omitted)
                  1.female |  -.0724031   .0251082    -2.88   0.004    -.1216143   -.0231919
                           |
            female#c.age16 |
                        1  |   .0039313   .0077485     0.51   0.612    -.0112554    .0191181
                           |
    female#c.age16#c.age16 |
                        1  |   .0148111   .0063908     2.32   0.020     .0022854    .0273367
                           |
                expochange |   .4421352    .035928    12.31   0.000     .3717175    .5125529
                1.minority |   .0022791   .0273332     0.08   0.934     -.051293    .0558513
                           |
     minority#c.expochange |
                        1  |  -.3099835   .0741428    -4.18   0.000    -.4553007   -.1646663
                           |
                expochange |          0  (omitted)
                   incomem |   .0146007   .0048037     3.04   0.002     .0051855    .0240159
                           |
    c.expochange#c.incomem |  -.0478129   .0141375    -3.38   0.001     -.075522   -.0201038
                           |
                     _cons |   .5334629   .0180713    29.52   0.000     .4980438    .5688819
---------------------------+--------------------------------------------------------------
                   sigma_u |  .14136853
                   sigma_e |  .17125674
                       rho |  .40526199   (fraction of variance due to u_i)
------------------------------------------------------------------------------------------
```

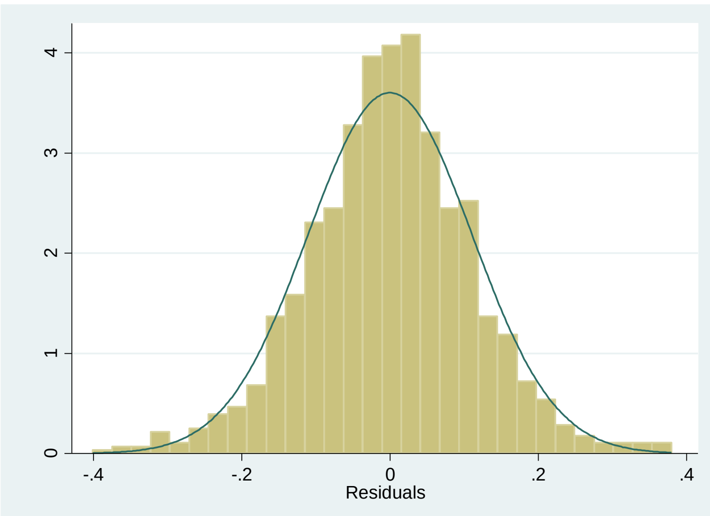
<u>Diagnostics for mixed effects models</u>

To conduct diagnostics for mixed effects models, we can obtain predicted values and various residuals; here is what we can obtain using predict command:

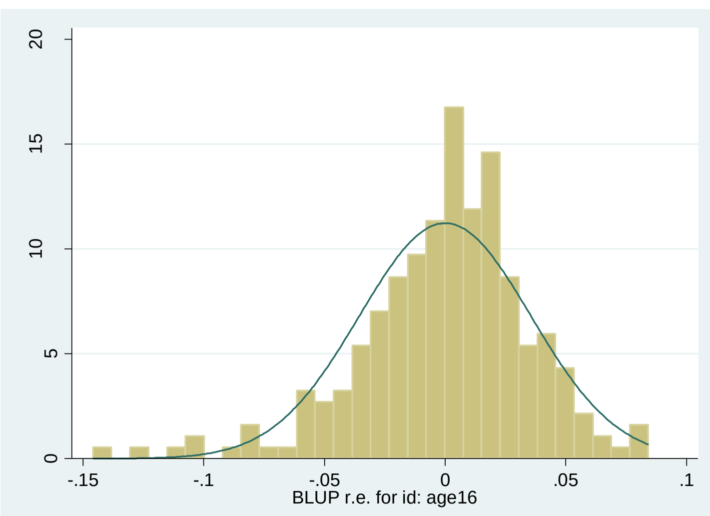
```
xb             xb, linear predictor for the fixed portion of the model
stdp           standard error of the fixed-portion linear prediction xb
fitted         fitted values, linear predictor of the fixed portion plus
                 contributions based on predicted random effects
residuals      residuals, response minus fitted values
rstandard      standardized residuals
reffects       best linear unbiased predictions (BLUPs) of the
               random effects. By default, BLUPs for all random effects in the
               model are calculated. You must specify q new variables, where q is
               the number of random-effects terms in the model.
reses           standard errors of the best linear unbiased predictions (BLUPs) of
               the random effects.  By default, standard errors for all BLUPs in
                the model are calculated. You must specify q new variables.
```

Thus, residuals will give you level 1 residuals, rstandard will give you standardized residuals, and reffects will give you level 2 residuals for each level 2 random component. You should examine both types of residuals to assess normality. For example, for a simple model without cross-level interactions:

```
. qui mixed attit c.age16##c.age16  expo  female minority income || id: 
c.age16##c.age16 expo, cov(unstructured)

. predict level1, resid
(139 missing values generated)

. predict level2*, reffects

. histogram level1, normal
(bin=30, start=-.40109414, width=.02602156)
```

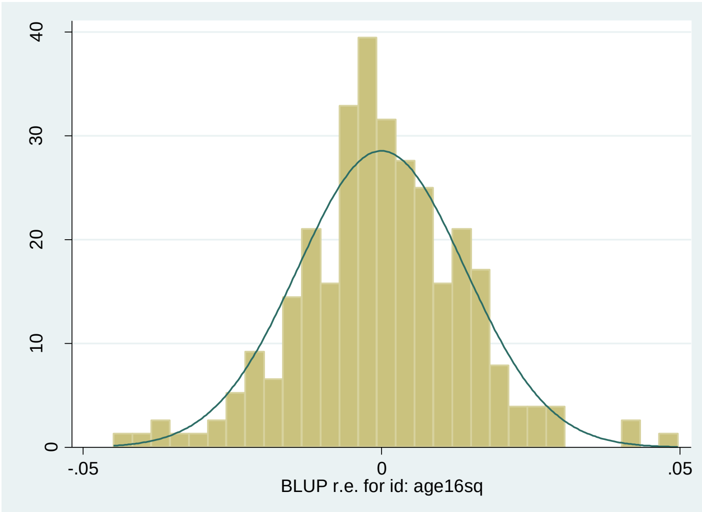



```
. egen tag=tag(id)

. histogram level21 if tag==1, normal
(bin=30, start=-.14591008, width=.00767065)
```

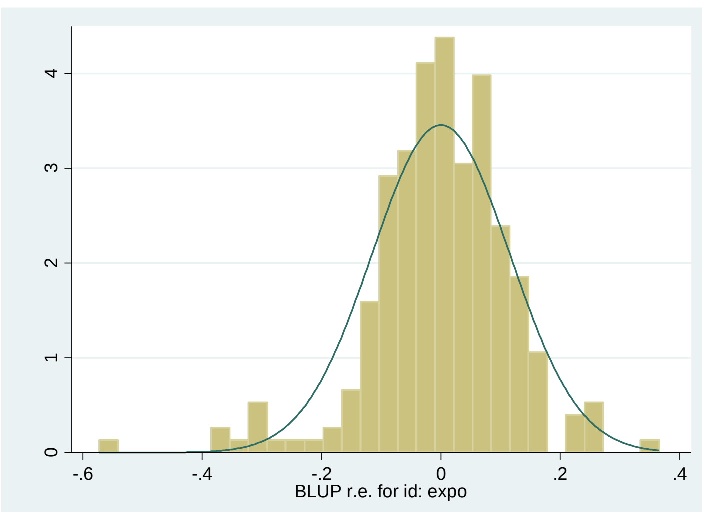



```
. histogram level22 if tag==1, normal
```

```
(bin=30, start=-.04489113, width=.00315281)
```




```
. histogram level23 if tag==1, normal
(bin=30, start=-.57256621, width=.03127522)
```

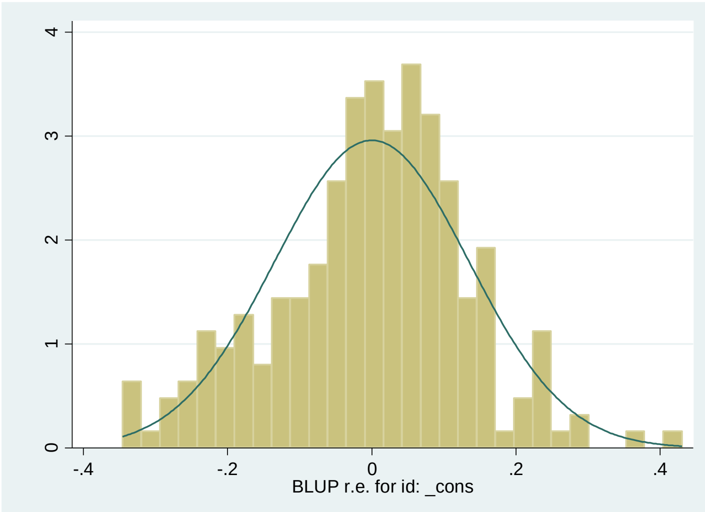



```
. histogram level24 if tag==1, normal
(bin=30, start=-.34564272, width=.02585592)
```

To restrict diagnostics for level 2 to one observation per person, you can either use a condition like tag==1 or you can create a separate level 2 dataset and conduct level 2 diagnostics there (do it after generating residual variables). But don't forget to also restrict cases to one per level 2 unit when doing preliminary data examination – that stage is also important! To create level 2 dataset:

```
. keep id attit female minority income incomem expomean

. bysort id: egen attitmean=mean(attit)

. drop attit

. bysort id: keep if _n==_N
(964 observations deleted)
```

```
. sum id
    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
          id |        241    900.1701    484.3652          5       1717
```



To assess linearity, you can plot residuals (level 1 and level 2) against each predictor using lowess (there should be no relationship between them). You can also examine residuals for potential outliers (i.e., look for observations with large residuals).

You can use robust option available with mixed for robust standard errors to obtain standard errors and significance tests that are less dependent on assumptions; you can also use bootstrapping, although it does take time to calculate (and we need to clear away xtset first, or we will get an error message). E.g.:

```
. xtset, clear

. bootstrap, cluster(id): mixed attit c.age16##c.age16##i.female
c.age16##c.age16##c.expomean c.expochange##c.income1 c.expochange##i.minority || id:
age16 age16sq expochange, cov(unstr)
(running mixed on estimation sample)

Bootstrap replications (50)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241
                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(13)     =     813.18
Log likelihood =  240.81527                     Prob > chi2       =     0.0000
                                        (Replications based on 241 clusters in id)
------------------------------------------------------------------------------
                     |   Observed   Bootstrap                         Normal-based
               attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------+--------------------------------------------------------------
               age16 |   .0448121    .0110298     4.06   0.000     .0231941    .0664301
                     |
```

```
      c.age16#c.age16 |  -.0071441   .0062649    -1.14   0.254    -.0194232    .0051349
                      |
             1.female |  -.0738328   .0170516    -4.33   0.000    -.1072533   -.0404122
                      |
       female#c.age16 |
                    1 |   .0039725   .0083453     0.48   0.634    -.0123841     .020329
                      |
female#c.age16#c.age16 |
                    1 |   .0151555   .0052815     2.87   0.004      .004804    .0255069
                      |
                age16 |          0  (omitted)
             expomean |   .6318958   .0420577    15.02   0.000     .5494642    .7143275
                      |
   c.age16#c.expomean |  -.0408274   .0169597    -2.41   0.016    -.0740677   -.0075871
                      |
      c.age16#c.age16#|
           c.expomean |  -.0091057   .0102475    -0.89   0.374    -.0291903     .010979
                      |
           expochange |   .5894723   .0742479     7.94   0.000     .4439492    .7349955
              income1 |   .0152012   .0037713     4.03   0.000     .0078096    .0225928
                      |
c.expochange#c.income1 |  -.0550901   .0166698    -3.30   0.001    -.0877623   -.0224178
                      |
           expochange |          0  (omitted)
           1.minority |   .0038862   .0259335     0.15   0.881    -.0469426    .0547151
                      |
 minority#c.expochange |
                    1 |  -.3121521   .1116763    -2.80   0.005    -.5310336   -.0932706
                      |
                _cons |     .13292   .0354378     3.75   0.000     .0634632    .2023768
--------------------------------------------------------------------------------------
------------------------------------------------------------------------------
                             |   Observed   Bootstrap         Normal-based
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0026167   .0004469      .0018723    .0036569
                var(age16sq) |   .0005755   .0001157       .000388    .0008536
               var(expoch~e) |   .0574034    .015231      .0341261    .0965581
                  var(_cons) |    .024968    .002662      .0202597    .0307706
          cov(age16,age16sq) |  -.0003702   .0001412     -.0006469   -.0000934
         cov(age16,expoch~e) |  -.0030557    .001399     -.0057977   -.0003137
            cov(age16,_cons) |   .0013106   .0005887      .0001568    .0024644
       cov(age16sq,expoch~e) |   .0015322   .0007393      .0000832    .0029811
          cov(age16sq,_cons) |  -.0015776   .0005165       -.00259   -.0005652
         cov(expoch~e,_cons) |   .0017044   .0032579      -.004681    .0080897
-----------------------------+------------------------------------------------
               var(Residual) |   .0191298   .0027874      .0143774    .0254531
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 278.16               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```