We continue working with High School and Beyond data (included with HLM software – see HLM folder → Examples →Chapter 2, data files: HSB1.sav and HSB2.sav).

After estimating a null model and assuring that we observe a significant amount of group-level variance, we proceed to build a multilevel explanatory model. A typical approach is to build such a model from bottom up.

Model 1. Conditional model with random intercept (one way ANCOVA with random intercept)

LEVEL 1 MODEL

$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij}) + r_{ij}$

LEVEL 2 MODEL

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10}$

MIXED MODEL

$\text{MATHACH}_{ij} = \gamma_{00} + \gamma_{10} *\text{SES}_{ij} + u_{0j} + r_{ij}$

```
Sigma_squared =      37.03440

 Tau
 INTRCPT1,B0      4.76815

Tau (as correlations)
 INTRCPT1,B0  1.000


 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                       0.843
 ----------------------------------------------------

The value of the likelihood function at iteration 6 = -2.332167E+004

 The outcome variable is  MATHACH
 Final estimation of fixed effects:
```

 The outcome variable is MATHACH
 Final estimation of fixed effects:

		Standard		Approx.	
Fixed Effect	Coefficient	Error	T-ratio	d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	12.657481	0.187984	67.333	159	0.000
For SES slope, B1					
INTRCPT2, G10	2.390199	0.105719	22.609	7183	0.000

```
 The outcome variable is  MATHACH
 Final estimation of fixed effects
 (with robust standard errors)
```

```
-------------------------------------------------------------------------------
                                    Standard               Approx.
   Fixed Effect            Coefficient   Error    T-ratio   d.f.    P-value
-------------------------------------------------------------------------------
For         INTRCPT1, B0
   INTRCPT2, G00          12.657481  0.187330   67.568      159    0.000
For        SES slope, B1
   INTRCPT2, G10           2.390199  0.119309   20.034     7183    0.000
-------------------------------------------------------------------------------


Final estimation of variance components:
-------------------------------------------------------------------------------
Random Effect             Standard    Variance    df    Chi-square  P-value
                          Deviation   Component
-------------------------------------------------------------------------------
INTRCPT1,     U0           2.18361      4.76815   159   1037.09077   0.000
 level-1,     R            6.08559     37.03440
-------------------------------------------------------------------------------


Statistics for current covariance components model
--------------------------------------------------
Deviance                      = 46643.331427
Number of estimated parameters = 2
```

Note that we now estimate two fixed effects – the intercept and the effect of student's SES. The intercept γ_{00} is no longer the average math achievement – it is now math achievement for someone with all predictors equal to zero. In this case, it's math achievement for someone with SES=0, but because the SES scale was designed to have a mean of 0, the intercept (12.66) is essentially the math achievement for someone with average SES. The effect of SES, γ_{10}, can be interpreted as follows: one unit increase in SES is associated with 2.39 unit increase in one's math achievement. So math achievement for someone with SES being 1 unit above the mean would be:
12.66+2.39=15.05

Note that each β_{0j} is now the mean outcome for each group (i.e. school) adjusted for the differences among these groups in SES.

As we now accounted for some portion of the variance by controlling for SES, we can calculate the adjusted intra-class correlation: ρ=4.76815/(4.76815+37.03440)= .11406362

The decrease in ρ from .18035673 to .11406362 reflects a reduction in the relative share of between-school variance when we control for student SES. But there is still significant variation across schools.

We could also calculate the proportion of variance explained at each level by comparing the current variance estimates to those in the null model. (This is the easiest method recommended by Bryk and Raudenbush; another method is suggested by Snijders and Bosker; you can see their book for more details):

(8.61431 - 4.76815)/8.61431 = .44648498

(39.14831 - 37.03440)/ 39.14831 = .05399748

So controlling for individuals' SES explained 45% of between-school variance, and 5% of within-school variance in math achievement. We could also calculate the total percentage of variance explained:
(39.14831+8.61431-4.76815-37.03440)/(39.14831+8.61431)= .12478524
So students' SES explained 12% of the total variance in math achievement.

Let's take this one step further. So far we assumed that an individual student's SES would have the same impact on his or her math achievement regardless of the school where that student is studying. Let's relax that assumption.

Model 2. Model with random intercept and random slopes (one way ANCOVA with random intercept and slopes)

LEVEL 1 MODEL

$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij}) + r_{ij}$

LEVEL 2 MODEL

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

Here, level-1 slopes are allowed to vary across level-2 units. But we do not try to predict that variation – only describe it.

Now we have:
γ_{00} is the average intercept across the level-2 units (grand mean of math achievement controlling for SES – i.e. the mean for someone with SES=0)
γ_{10} is the average SES slope across the level-2 units (i.e. average effect of SES across schools)
u_{0j} is the unique addition to the intercept associated with level-2 unit j (indicates how the intercept for school j differs from the grand mean)
u_{1j} is the unique addition to the slope associated with level-2 unit j (indicates how the effect of SES in school j differs from the average effect of SES for all schools)

Note that:
$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N\left(0, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right)$$

Our tau matrix now contains the variance in the level-1 intercepts (τ_{00}), the variance in level-1 slopes (τ_{11}), as well as the covariance between level-1 intercepts and slopes ($\tau_{01} = \tau_{10}$). Note that covariance value indicates how much intercepts and slopes covary: in our example (below), there is a negative correlation between intercepts and slopes. That is, the higher the intercept, the smaller the slope (i.e. if the school level of math achievement is high, the effect of SES in that school is smaller).

```
Sigma_squared =      36.82835

 Tau
 INTRCPT1,B0     4.82978       -0.15399
     SES,B1     -0.15399        0.41828
```

```
Tau (as correlations)
 INTRCPT1,B0  1.000 -0.108
      SES,B1 -0.108  1.000


  ----------------------------------------------------
   Random level-1 coefficient   Reliability estimate
  ----------------------------------------------------
   INTRCPT1, B0                     0.797
       SES, B1                      0.179
  ----------------------------------------------------
The value of the likelihood function at iteration 21 = -2.331928E+004

  The outcome variable is  MATHACH

  Final estimation of fixed effects:
  --------------------------------------------------------------------------------
                                       Standard             Approx.
     Fixed Effect          Coefficient Error     T-ratio    d.f.    P-value
  --------------------------------------------------------------------------------
For        INTRCPT1, B0
    INTRCPT2, G00           12.664935  0.189874  66.702      159    0.000
For       SES slope, B1
    INTRCPT2, G10            2.393878  0.118278  20.240      159    0.000
  --------------------------------------------------------------------------------


  The outcome variable is  MATHACH

  Final estimation of fixed effects
  (with robust standard errors)
  --------------------------------------------------------------------------------
                                       Standard             Approx.
     Fixed Effect          Coefficient Error     T-ratio    d.f.    P-value
  --------------------------------------------------------------------------------
For        INTRCPT1, B0
    INTRCPT2, G00           12.664935  0.189251  66.921      159    0.000
For       SES slope, B1
    INTRCPT2, G10            2.393878  0.117697  20.339      159    0.000
  --------------------------------------------------------------------------------


  Final estimation of variance components:
  --------------------------------------------------------------------------------
Random Effect           Standard    Variance   df    Chi-square  P-value
                        Deviation   Component
  --------------------------------------------------------------------------------
INTRCPT1,      U0        2.19768      4.82978   159    905.26472   0.000
    SES slope, U1        0.64675      0.41828   159    216.21178   0.002
 level-1,      R         6.06864     36.82835
  --------------------------------------------------------------------------------


  Statistics for current covariance components model
  --------------------------------------------------------
Deviance                    = 46638.560929
Number of estimated parameters = 4
```

Here, like in the previous model, the math achievement for someone with average SES (SES=0) is 12.66; each unit increase in SES is associated with 2.39 units increase in math achievement. But, examining variance components, we notice that there is a significant variation in slopes (p-value =.002) – this means that SES effects vary across schools, so 2.39 is the effect for an average school. Here, if we want to divide the unexplained variance into within-school and between-school, we need to take into account the covariance: level 1 component is simply 36.82835, but level 2 component is (4.82978+0.41828+2*-0.15399)= 4.94008.

Note that in addition to the average reliability of school means, we now also have an estimate of reliability for the effect of SES, and it is much lower: .179.   It is normal that the reliability of slopes is much lower than that of intercepts. The precision of estimation of the intercept (which in this case is a school mean) depends only on the sample size within each school. The precision of estimation of the slope depends both on the sample size and on the variability of SES within that school. Schools that are homogeneous with respect to SES will exhibit slope estimation with poor precision. But the average reliability of the slopes is relatively low because the true slope variance across schools is much smaller than the variance of the true means.

Note that low reliabilities do not invalidate the HLM analysis, but very low reliabilities (typically < .10) often indicate that a random coefficient might be considered fixed (i.e., the same across groups) in subsequent analyses.

Model 3. Means-as-outcomes model (a.k.a. Intercepts as outcomes)

LEVEL 1 MODEL

$$\text{MATHACH}_{ij} = \beta_{0j} + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$$

This model allows us to predict variation in the levels of math achievement using level-2 variables. If we would attempt to do this using regular OLS, we would be artificially inflating the sample size and pretend we have 7185 data points to evaluate the effect of type of school (Catholic vs public), when in fact it's only 160 schools. Aggregating the data to school level would be more acceptable, but we would not have any assessment of within-school variation. Note, however, that the sample size for level 2 becomes important as soon as you try to include predictors at this level!

```
Sigma_squared =      39.15135

 Tau
 INTRCPT1,B0      6.67771

Tau (as correlations)
 INTRCPT1,B0  1.000


 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
   INTRCPT1, B0                     0.877
 ----------------------------------------------------
The value of the likelihood function at iteration 4 = -2.353915E+004

 The outcome variable is  MATHACH
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                  Standard          Approx.
    Fixed Effect         Coefficient  Error     T-ratio   d.f.    P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00          11.393043  0.292887   38.899     158     0.000
       SECTOR, G01          2.804889  0.439142    6.387     158     0.000
```

```
The outcome variable is  MATHACH
Final estimation of fixed effects
(with robust standard errors)
----------------------------------------------------------------------------
                                        Standard              Approx.
    Fixed Effect           Coefficient  Error      T-ratio    d.f.    P-value
----------------------------------------------------------------------------
For       INTRCPT1, B0
   INTRCPT2, G00            11.393043   0.292258    38.983    158     0.000
     SECTOR, G01             2.804889   0.435823     6.436    158     0.000
----------------------------------------------------------------------------


Final estimation of variance components:
----------------------------------------------------------------------------
Random Effect             Standard     Variance    df    Chi-square  P-value
                          Deviation    Component
----------------------------------------------------------------------------
INTRCPT1,      U0          2.58413       6.67771   158   1296.76559   0.000
 level-1,      R           6.25710      39.15135
----------------------------------------------------------------------------


Statistics for current covariance components model
--------------------------------------------------
Deviance                      = 47078.295826
Number of estimated parameters = 2
```

Here, we see a positive effect of Catholic schools on math achievement – the average achievement of Catholic schools is 2.8 units higher than for public schools. The intercept now is an average value for a public school student. There is, nevertheless, significant school-level variance remaining. As we did with earlier models, we can calculate the percentage of variance in math achievement explained by school type. Note that here we only explain level 2 variance – level 1 variance remained the same. For level 2 variance:

(8.61431 - 6.67771)/8.61431 = .22481197

So 22% of school-level variance in math achievement was explained by type of school.

Model 4. Means as outcomes model with level 1 covariate

As a next step, we can add level-1 covariates to this means-as-outcomes model. These level-1 variables can be added as fixed effects (i.e., assuming that the effects of these covariates are the same for all schools –that's what we did in model 1) or as random effects (i.e., assuming that the effects of level 1 variables vary across schools – that's what we did in model 2). We will right away opt for a more complex option, assuming that the effects of level 1 variable – SES – vary across schools.

LEVEL 1 MODEL

$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij}) + r_{ij}$

LEVEL 2 MODEL

$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

```
Sigma_squared =      36.79508
```

```
   Tau
    INTRCPT1,B0      3.96459         0.71641
         SES,B1      0.71641         0.44990

   Tau (as correlations)
    INTRCPT1,B0  1.000  0.536
         SES,B1  0.536  1.000


         ----------------------------------------------------
          Random level-1 coefficient   Reliability estimate
         ----------------------------------------------------
          INTRCPT1, B0                          0.765
              SES, B1                           0.189
         ----------------------------------------------------


   The value of the likelihood function at iteration 21 = -2.330093E+004
    The outcome variable is  MATHACH

    Final estimation of fixed effects:
    ----------------------------------------------------------------------------
                                        Standard            Approx.
        Fixed Effect          Coefficient  Error    T-ratio  d.f.    P-value
    ----------------------------------------------------------------------------
   For        INTRCPT1, B0
       INTRCPT2, G00          11.476646  0.231587   49.557    158     0.000
         SECTOR, G01           2.533835  0.344798    7.349    158     0.000
   For      SES slope, B1
       INTRCPT2, G10           2.385451  0.118329   20.160    159     0.000
    ----------------------------------------------------------------------------


    The outcome variable is  MATHACH

    Final estimation of fixed effects
    (with robust standard errors)
    ----------------------------------------------------------------------------
                                        Standard            Approx.
        Fixed Effect          Coefficient  Error    T-ratio  d.f.    P-value
    ----------------------------------------------------------------------------
   For        INTRCPT1, B0
       INTRCPT2, G00          11.476646  0.225026   51.001    158     0.000
         SECTOR, G01           2.533835  0.352411    7.190    158     0.000
   For      SES slope, B1
       INTRCPT2, G10           2.385451  0.119008   20.044    159     0.000
    ----------------------------------------------------------------------------

    Final estimation of variance components:
    ----------------------------------------------------------------------------
    Random Effect         Standard    Variance    df    Chi-square  P-value
                          Deviation   Component
    ----------------------------------------------------------------------------
    INTRCPT1,       U0     1.99113      3.96459   158    766.83844    0.000
         SES slope, U1     0.67075      0.44990   159    216.12223    0.002
     level-1,       R      6.06589     36.79508
    ----------------------------------------------------------------------------


    Statistics for current covariance components model
    --------------------------------------------------
    Deviance                      = 46601.861400
    Number of estimated parameters = 4
```

Now the intercept is the value for average SES student in a public school: 11.48. The value for an average-SES Catholic school student is 2.53 units higher: 11.45+2.53=13.98
Further, one unit increase in SES is associated with 2.39 units increase in math score. But there is still significant variation across schools in intercepts, and there is also significant variation in SES slopes – so SES doesn't have the same effect across schools.

Model 5. Intercepts and Slopes as outcomes (a.k.a. Cross-level Interactions model)

Next, we will try to explain this variation in SES effects across schools – we'll explore whether this variation can be attributed to the type of school – public vs Catholic.

LEVEL 1 MODEL

$$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{SECTOR}_j) + u_{1j}$$

This type of model allows us to explain the variation in both intercepts and slopes. Sometimes, it's called cross-level interactions model because we make the effect of level-1 variables (SES) dependent upon the value of level-2 variables (in this case, SECTOR).

```
Sigma_squared =      36.76311

 Tau
 INTRCPT1,B0      3.83295         0.54112
      SES,B1      0.54112         0.12988

Tau (as correlations)
 INTRCPT1,B0  1.000  0.767
      SES,B1  0.767  1.000


 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                      0.759
      SES, B1                       0.064
 ----------------------------------------------------
The value of the likelihood function at iteration 198 = -2.328373E+004

 The outcome variable is  MATHACH
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                   Standard            Approx.
    Fixed Effect         Coefficient  Error    T-ratio  d.f.    P-value
 ----------------------------------------------------------------------------
For        INTRCPT1, B0
    INTRCPT2, G00        11.750237  0.232241   50.595    158     0.000
      SECTOR, G01         2.128611  0.346651    6.141    158     0.000
For      SES slope, B1
    INTRCPT2, G10         2.958798  0.145460   20.341    158     0.000
      SECTOR, G11        -1.313096  0.219062   -5.994    158     0.000
 ----------------------------------------------------------------------------

 The outcome variable is  MATHACH
 Final estimation of fixed effects
```

```
(with robust standard errors)
-------------------------------------------------------------------------
                                    Standard              Approx.
    Fixed Effect          Coefficient  Error    T-ratio   d.f.     P-value
-------------------------------------------------------------------------
For         INTRCPT1, B0
    INTRCPT2, G00         11.750237  0.218675   53.734      158    0.000
      SECTOR, G01          2.128611  0.355697    5.984      158    0.000
For       SES slope, B1
    INTRCPT2, G10          2.958798  0.144092   20.534      158    0.000
      SECTOR, G11         -1.313096  0.214271   -6.128      158    0.000
-------------------------------------------------------------------------
Final estimation of variance components:
-------------------------------------------------------------------------
Random Effect          Standard     Variance    df    Chi-square  P-value
                       Deviation    Component
-------------------------------------------------------------------------
INTRCPT1,       U0       1.95779      3.83295    158   756.04082    0.000
     SES slope, U1       0.36039      0.12988    158   178.09113    0.131
 level-1,        R       6.06326     36.76311
-------------------------------------------------------------------------


Statistics for current covariance components model
-------------------------------------------------
Deviance                    = 46567.464841
Number of estimated parameters = 4
```

In terms of fixed effects, the difference between this model and the previous one is the introduction of the effect of SECTOR on SES, which can be interpreted as an interaction term between SECTOR and SES. That is, the effect of SES for public schools is 2.96 per one unit increase in SES; but for Catholic schools, the effect of SES is (2.96-1.31)=1.65 per one unit increase in SES. So students' math scores are more sensitive to their SES in public schools than in Catholic schools.

Significance tests tell us that the effect of SES is significant in public schools (that's significance test for G10), and that SES effect is significantly different in Catholic vs public schools (that's significance test for G11), but how do we find out if SES has a significant effect on math achievement in Catholic schools? That is, how do we know if that 1.65 is significantly different from zero? To answer that question, we would need to calculate a significance test for that coefficient. For that, we should first calculate the standard error of this so-called "simple slope" using the following formula:

$$S_{b(X \text{ at } Y=Z)} = \text{sqrt}[S^2_{bXmain} + 2\,S^2_{bXmain\_bXY} + (Z)^2 S^2_{bXY}]$$

where S^2_{bXmain} is the squared standard error of the main effect of X, S^2_{bXY} is the squared standard error of the interaction term between X and Y, and $S^2_{bXmain\_bXY}$ is the covariance of the two (main effect and interaction); this covariance can be obtained from the covariance matrix of regression coefficients that can be generated by selecting Other Settings → Output Settings and checkmarking the corresponding box. When we run the model after that, we get the additional message:

```
tauvc.dat, containing tau has been created.
gamvc.dat, containing the variance-covariance matrix of gamma has been
created.
gamvcr.dat, containing the robust variance-covariance matrix of gamma has
```

```
been created.
```

When we open gamvc.dat, we see
```
     11.7506613        2.1284225        2.9587976        -1.3130961
 5.3938317E-002 -5.3938317E-002  9.1020632E-003 -9.1020632E-003
-5.3938317E-002  1.2017077E-001 -9.1020632E-003  1.3365313E-002
 9.1020632E-003 -9.1020632E-003  2.1158704E-002 -2.1158704E-002
-9.1020632E-003  1.3365313E-002 -2.1158704E-002  4.7987956E-002
```

The first row lists coefficients themselves (G00, G01, G10, G11) in order to label the columns. Therefore, S^2_{bXmain} is the squared standard error of the main effect of SES (G10) = `2.1158704E-002` = .021158704. S^2_{bXmain} is the squared standard error of the interaction term (G11) = `4.7987956E-002` = .047987956. $S^2_{bXmain\_bXY}$ is the covariance of the two = `-2.1158704E-002` = -.021158704.

In this case, Z=1 because the difference between public (0) and Catholic (1) is 1, but if the moderator variable is continuous, Z can be something else; for example, the standard deviation of Y. So $S_{b(X \text{ at } Y=Z)}$ = sqrt(.021158704+2*-.021158704 +1*.047987956) = .16379637

Therefore, t-ratio: 1.65/.164=10.06, which is well above the cutoff for p<.001 for large samples (t=3.29). Therefore, the simple slope of SES at SECTOR=1 is statistically significant.

Preacher and colleagues present an online calculator to compute the tests of simple slopes at http://www.quantpsy.org/interact/hlm2.htm

Our case is case 3 as we have a cross-level interaction; note that the order of gammas in the calculator is different from the order in the output. After entering all the values, we get:

```
    CASE 3 TWO-WAY INTERACTION SIMPLE SLOPES OUTPUT

Your Input
========================================================
  w1(1)        = 0
  w1(2)        = 1
  Intercept    = 11.7506613
  x1 Slope     = 2.9587976
  w1 Slope     = 2.1284225
  w1x1 Slope   = -1.3130961
  alpha        = 0.05
  df(int)      = 158
  df(slp)      = 158

Asymptotic (Co)variances
========================================================
  var(g00) 0.05393832
  var(g10) 0.0211587
  var(g01) 0.12017077
  var(g11) 0.04798796
  cov(g00,g01) -0.05393832
  cov(g10,g11) -0.0211587
  cov(g00,g10) 0.00910206
  cov(g01,g11) 0.01336531
```

```
Region of Significance on w (level-2 predictor)
========================================================
  w1 at lower bound of region = 1.7821
  w1 at upper bound of region = 3.166
  (simple slopes are significant *outside* this region.)

Simple Intercepts and Slopes at Conditional Values of w
========================================================
  At w1(1)...
    simple intercept = 11.7507(0.2322), t=50.5957, p=0
    simple slope     = 2.9588(0.1455), t=20.3409, p=0
  At w1(2)...
    simple intercept = 13.8791(0.2574), t=53.9294, p=0
    simple slope     = 1.6457(0.1638), t=10.0472, p=0

Simple Intercepts and Slopes at Region Boundaries for w
========================================================
  Lower Bound...
    simple intercept = 15.5437(0.4933), t=31.51, p=0
    simple slope     = 0.6187(0.3133), t=1.975, p=0.05
  Upper Bound...
    simple intercept = 18.4892(0.9576), t=19.3085, p=0
    simple slope     = -1.1985(0.6068), t=-1.9751, p=0.05
```

The relevant information is:
```
    simple slope     = 1.6457(0.1638), t=10.0472, p=0
```

Next, going back to HLM output, we can also examine the amount of variance in SES slopes explained by SECTOR: the unconditional variance in SES slopes was 0.44990, and the variance in this model (controlling for SECTOR) is only 0.12988.

(0.44990-0.12988)/0.44990 = .71131363

So SECTOR explained 71% of between-school variance in effects of SES on math achievement. Also note that now that we controlled for sector, the variation in SES slopes across schools is no longer significant. Therefore, we could run this model as a model with nonrandomly varying slopes.

Model 6. Model with Nonrandomly Varying Slopes.

LEVEL 1 MODEL

$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}(\text{SES}_{ij}) + r_{ij}$

LEVEL 2 MODEL

$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SECTOR}_j) + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{SECTOR}_j)$

```
 Sigma_squared =     36.84019

 Tau
 INTRCPT1,B0     3.69423
```

```
Tau (as correlations)
 INTRCPT1,B0  1.000

 --------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 --------------------------------------------------
  INTRCPT1, B0                       0.808
 --------------------------------------------------
The value of the likelihood function at iteration 6 = -2.328616E+004

 The outcome variable is  MATHACH
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                     Standard           Approx.
     Fixed Effect        Coefficient Error     T-ratio  d.f.      P-value
 ----------------------------------------------------------------------------
 For        INTRCPT1, B0
    INTRCPT2, G00          11.797994  0.228514  51.629    158     0.000
      SECTOR, G01           2.138170  0.341344   6.264    158     0.000
 For      SES slope, B1
    INTRCPT2, G10           2.951177  0.140609  20.989   7181     0.000
      SECTOR, G11          -1.312849  0.211996  -6.193   7181     0.000
 ----------------------------------------------------------------------------


 The outcome variable is  MATHACH
 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                     Standard           Approx.
     Fixed Effect        Coefficient Error     T-ratio  d.f.      P-value
 ----------------------------------------------------------------------------
 For        INTRCPT1, B0
    INTRCPT2, G00          11.797994  0.214976  54.880    158     0.000
      SECTOR, G01           2.138170  0.350028   6.109    158     0.000
 For      SES slope, B1
    INTRCPT2, G10           2.951177  0.143779  20.526   7181     0.000
      SECTOR, G11          -1.312849  0.212824  -6.169   7181     0.000
 ----------------------------------------------------------------------------


 Final estimation of variance components:
 ----------------------------------------------------------------------------
 Random Effect           Standard   Variance   df    Chi-square  P-value
                         Deviation  Component
 ----------------------------------------------------------------------------
 INTRCPT1,    U0          1.92204     3.69423  158    837.19099   0.000
  level-1,    R           6.06961    36.84019
 ----------------------------------------------------------------------------


 Statistics for current covariance components model
 --------------------------------------------------
 Deviance                    = 46572.326387
 Number of estimated parameters = 2
```

Note that we are able to model how sector shapes SES, but we do not allow any other variation in SES slopes because there is no significant variation beyond that accounted for by sector.